THE CLINICAL USE OF PRISMS

MADDOX

FIFTH EDITION

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OPHTHALMOLOGICAL PRISMS.

THE CLINICAL USE OF PRISMS;

AND THE

Decentring of Lenses.

BY

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PREFACE TO THE FIFTH EDITION.

THAT after eighteen years this little book should be read still is "labour well rewarded," for many new contributions in the first edition must now appear familiar and even elementary.

To this edition has been added an account of some new instruments, the prism-verger, the duant, the spirit-level prismeter, and a means of measuring hyperphoria in near vision.

Moreover, since workmen must sometimes be puzzled by the diverse modes of prescribing prisms, a "workman's page" has been placed in in the Appendix. It would be well if methods of prescribing were reduced to two in number, namely, (*a*) by degrees of deviation, and (*b*) by percentage of deviation.

No metric unit for prisms had been proposed at the time of the first edition, and the able work in this direction, of Prentice, of Swan Burnett, and of Dennett, deserves our thanks.

It is an undoubted advantage to be able to define angles not only in terms of the circumference, i.e., in degrees, but also in terms of the

radius, and this is the aim of the prism-dioptre and the centrad. Each of these is, however, too cramped, since the former does not permit of multiplication, nor the latter of measuring on the straight, and the proposal in this volume is to combine the advantages of both, and designate prisms simply by their *percentage* of deviation. This conveys its own meaning to the mind, while enabling measurement to be specified in a manner suited to the occasion.

Manufacturers prefer to measure along a tangent-line; where multiplication is involved, it is better to measure along the arc-line; for accurate study of metre-angles it should be along the sine-line; while for decentration, if treated with rigid accuracy, taking account of spherical aberration, the measurement would be along a curved line which agrees with none of these, but which could be plotted out. The name "centune" (written as %) proposed in these pages enables advantage to be taken of each of these measurements accurately when appropriate, without binding to any when the difference is immaterial.

The author's thanks are due to his helpers, Drs. Trench and Yearsley, for literary aid in overlooking the proofs.

Some of the mathematical treatment of prisms has been relegated to the appendix, and in the hope that the book may be useful to the greater number, its arrangement is such that any reader who wishes to do so may skip formulæ and small print, without loss, it is hoped, in the understanding of what follows.

Bournemouth, 1907. E. E. M.

FROM PREFACE TO FIRST EDITION.

THE title of this little work indicates its chief features, but does not cover its ground completely. Its first object was to communicate a series of aids to precision in the use of prisms, worked out during several years, which it is hoped will be of some service in this difficult by-way of ophthalmic practice. They have, however, been introduced by a sketch of the simplest properties of prisms, and supplemented by a brief account of their chief clinical uses. . . . . It will not be supposed that the precision aimed at in these pages is necessary in the majority of refraction cases. It is chiefly anomalies of convergence, faulty tendencies of the ocular muscles, and the needs of the increasing neurasthenic class of patients that are kept in view. In these last cases, spectacles cause discomfort, however perfectly refraction may be corrected, unless the lenses are also suitably placed in respect of convergence, and sometimes even then, for a time.

Part of the manuscript was kindly read over by my friend, Dr. Poulett Wells, to whom I am indebted for one or two valuable corrections, and, indeed, for suggesting the need of a book on prisms.

July, 1889.

CONTENTS.

Contents

PRISMS

CHAPTER I

GENERAL PROPERTIES

THE prisms used in Ophthalmology are thin wedges of glass, having two plane polished faces set at a very acute angle.

The optical power of a prism depends both on the size of this angle and on the refractive index of the glass. Since the crown glass usually employed has a fairly constant refractive index, the angle between the faces, which varies in different prisms, is the feature which practically determines their strength. In contrast to the flint glass which, on account of its high dispersive power, is used for the manufacture of prisms intended for the production of a spectrum, that employed for ophthalmological prisms should separate or "disperse" the light of different wave-lengths as little as possible.

Since the terminology of prisms used in ophthalmology differs from that of geometry and physics, it will be well to notice some of the terms employed.

Refracting Surfaces.—This is the term used to denote the two plane faces mentioned above. By "refraction" is meant the deflection, or sudden bending from its former course, of light as it enters (other than perpendicularly) the surface of a rarer or denser medium. The interior of a prism does not refract light, since its rays travel through the glass in straight lines. It is only as light enters or leaves the prism, and thus passes from one medium to another, that it undergoes refraction. Herein lies the reason that the two plane polished faces are called the "refracting surfaces." In *Fig.* 1 they are represented by the faces *a c f b* and *a d c b*.

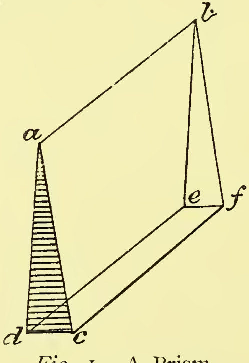

Fig. 1.—A Prism.

Refracting Angle.—This is the wedge between the two refracting faces, and must be carefully distinguished from the angle of deviation which denotes the deflection experienced by light when it traverses a prism. It will be seen that the

refracting angle is a physical feature of the prism itself, while the deviating angle is that of the light which goes through it. The former angle is invariable for each prism, while the latter is greater for violet than for red light, and varies, too, with the incidence, i.e., the direction in which the light falls on the prism. It was perhaps for these reasons that Donders, who, after Krecke's initiative, was the first to introduce prisms into ophthalmology, selected the refracting angle of prisms for their notation. Thus, a No. 1 prism came to mean one with a refracting angle of one degree, a No. 2 prism one with a refracting angle of two degrees, and so on. To avoid confusion, it is better not to speak of a 1° or 2°, etc., prism when we refer to the refracting angle, lest it should be mistaken for the deviating angle, but to specify a No. 1 or No. 2 prism, etc. When the deviating angle is referred to, the letter *d* should be added for further distinction, according to Edward Jackson's recommendation. Thus, for example, a No. 5 prism specifies one with a refracting angle of 5° between its two plane faces, while a 5° *d* prism indicates one which deflects ordinary white light (passing through its faces symmetrically) by an angle of 5°. We shall see later that the deviating angle of an ordinary weak prism is approximately half the refracting angle, so that the same prism could be described as a No. 4, or as a 2° *d* prism.

Trial cases are still often provided with a series of prisms, marked in the old way, from No. 1 or No. 2 up to No. 12 or No. 24, but since the author gave to Messrs. Curry & Paxton, in 1886, a means of measuring the deviating angle of prisms, they have been able to make them marked according to their deviation of light.

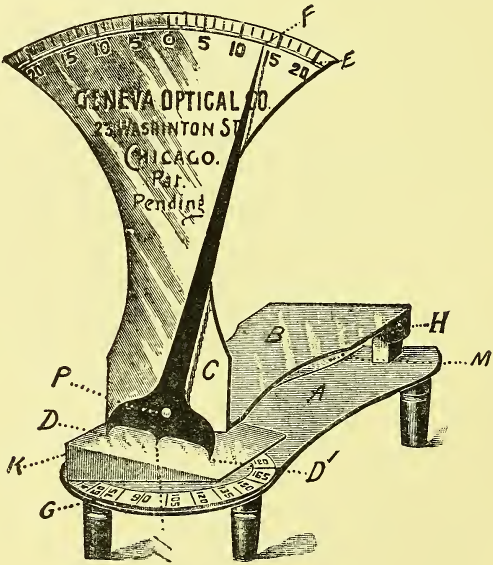

Fig. 2.—Prism Measure : the index points to the required angle when the teeth at its foot are pressed upon the prism.

It is to the ophthalmic surgeon that a knowledge of the deviating angle is of advantage. To the optical workman, on the contrary, the refracting angle is the only important item. He may first

set a pair of compasses so that their legs shall enclose the prescribed angle, and then shape and grind the piece of glass into a wedge that will just fit in between the two legs. To measure the refracting angle of a manufactured prism, we may resort to the same manœuvre, and after grasping the prism between a pair of compasses, measure the angle they include on a protractor. But if the prism be mounted in a metal ring, this method is impracticable, and the "prism measure" of the Geneva Optical Company (*Fig.* 2) will be found so useful as to merit description.

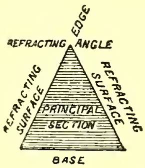

Fig. 3.—A prism in principal section.

From the base of a pointer project three teeth, of which the central one is shorter than the others. The pointer itself works on a pivot attached to a hinged piece of metal bent about its middle at an angle of 90°. On placing a prism under the teeth and pressing them against it, the pointer is deflected through an angle equal to the refracting angle of the prism, and which can be read off from a graduated arc.

Principal Section.—Any section made through a prism at right angles to its refracting surfaces is called a "principal section." Such a section is

shown in *Fig.* 3. The two refracting surfaces are seen to meet at the edge of the prism, enclosing between them the "refracting angle." The third side, opposite the edge, is called the base.

Rectangular Prisms.—In geometry, when we speak of a rectangular prism, we mean one which has a rectangular section. Ophthalmological prisms, however, are always triangular in section, as shown in *Fig.* 3, so that when we speak of them as "square," "rectangular," "circular," and so forth, it must be understood that these expressions apply only to the contour

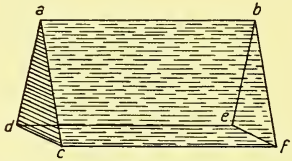

Fig. 4.—A rectangular prism.

of the refracting faces. It is evident that two plane faces may be of most varied outline, and yet be inclined to each other by the same angle, so that what we call the "shape" of an ophthalmological prism, in no way affects its strength. A square prism is represented in *Fig.* 1 and a rectangular one in *Fig.* 4. The "refracting surfaces" are lettered *a c f b* and *a d e b*, meeting along the "edge" *a b*, and enclosing the "refracting angle" *d a c*. The remaining surface *d e f c* is called the "base."

For some clinical purposes square prisms are extremely useful, since it is so easy to hold them in any desired position. Their disadvantage is that they will not fit into the ordinary trial-frame, but in every other respect their shape is preferable. It would be well for trial-cases to be provided with at least one square prism, having a deviating angle of about 6°. It will be found very useful in connection with the arrow scale for near vision shown in *Fig.* 45. A convenient way of using square prisms is that recommended by Noyes, in which a series of increasing strength are joined together in a frame, so that one after another can be made to pass before the eye. Such a series is shown in *Fig.* 5.

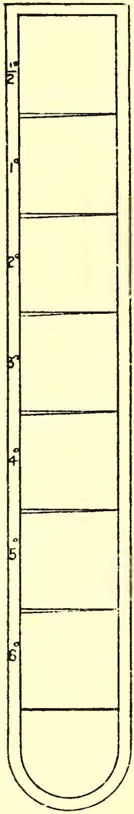

Fig. 5.—A "bar" of prisms (Gen. Opt. Co.)

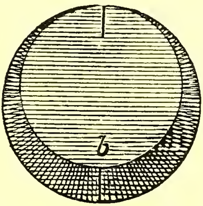

Fig. 6.—A circular prism.

Circular Prisms.—Prisms for a trial case are generally of a circular contour, to admit of their

insertion into the same trial-frame as the lenses. Such a prism is shown in *Fig.* 6, and the imaginary design in *Fig.* 7 is intended to show how a circular prism is related to a rectangular one, as though one were cut out of the other.

Apex and Base of Circular Prisms.—The thinnest part of a circular prism is called the *apex*. It is the point where the two refracting surfaces most nearly meet.

The *base* of a circular prism is the thickest part, exactly opposite the apex. It is not therefore a

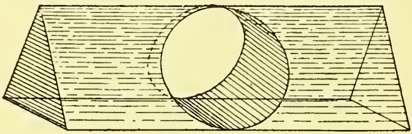

Fig. 7.—To show the relation of a circular prism to a rectangular one.

surface as in the case of a rectangular prism, but is indicated by a line (*b* in *Fig.* 6) joining the most widely separated points of the two circular faces.

An imaginary straight line from the apex to the centre of the base let us call the *base-apex line*. The reason for giving it a name is that all objects viewed through prisms appear displaced in a direction parallel to this line, as illustrated in *Fig.* 8, where a real object (*O*) appears displaced to *e* in a direction parallel to the base-apex line *p a*.

Edge of a Prism.—It is easy to understand what is the edge of a rectangular prism, for, as already explained, and shown in *Figs.* 4 and 7, it is the line *a b*, in which the refracting surfaces meet. But in a circular prism the surfaces meet only at a point. A circular prism, therefore, has no actual straight or knife edge, but only an imaginary one, as shown in *Fig.* 8 by *b c*, which passes through the apex at right angles to the base-apex line. It is the line in which the two refracting

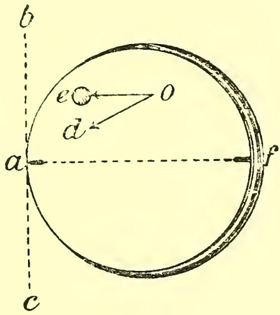

Fig. 8.—To show how an object at *O* appears displaced, not in the direction *d* towards the apex *a*, but in the direction *e* towards the edge *b c*, and parallel to the "base-apex line," *f a*.

surfaces *would* meet if they were prolonged, and it coincides, therefore, with the edge of the rectangular prism, out of which, as in *Fig.* 7, we may imagine the circular prism to have been cut. The name is taken from the edge of a knife in which the two polished faces of the blade meet. Though the knife edge of a circular prism has no visible existence, it has this significance, that

all objects seen through a prism appear displaced towards it. This optical edge must not be confused with the circular physical margin of the prism.

Prisms which are ground for insertion into spectacle frames have generally an oval contour. When this is the case, the thickest part of the prism is not always the base, nor is the thinnest part always the apex. This is shown in *Fig.* 9,

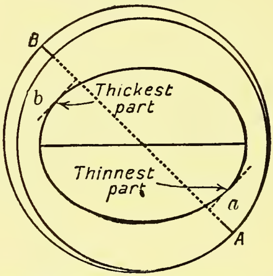

Fig. 9.—To show that in an oval prism cut obliquely from a circular one, the thickest and thinnest parts lie outside the base-apex line.

which makes clear this hitherto seemingly unnoticed fact. It represents a circular prism (with its apex at *A*, and its base at *B*), out of which an oval lens is cut. The thinnest part of the oval lens is at *a*, and the thickest part at *b*, points which are indicated by perpendiculars let fall from the base-apex line *A B* of the circular prism so as to meet the periphery of the oval prism tangentially. The touching points are the thinnest and thickest respectively.

Does this little anomaly introduce any difficulty in the true setting of an oval prism ? The answer is. Not if we employ any *optical* method of ascertaining the true base-apex line, nor yet if we use such an instrument as the prism measure of the Geneva Optical Company (*Fig.* 2). On placing any oval prism under its teeth and rotating the prism until the index records its maximum excursion, the teeth lie on the line *A B*, or on one parallel to it, thus indicating the direction of the true optical base-apex line, which is, of course, *A B*, and not *a b*.

CHAPTER II

SIMPLEST OPTICAL PROPERTIES

Refraction by Prisms.—In the case of a well-made clinical prism, we have only two media to take account of (both of which are practically homogeneous and isotropic), viz., air and glass.

Good optical glass is a practically homogeneous medium ; that is, it is of equal density throughout. It is also an isotropic medium ; that is, of equal elasticity in all directions, so that when a ray of light enters, it continues as a single ray after refraction.

A doubly refracting, or anisotropic, medium, on the contrary, possesses the property of dividing a ray of light when incident from certain directions into two rays, one being refracted in a slightly different direction to the other. Rock crystal is an instance; and though it is possible to cut weak prisms out of this material in such a way that the refracting surfaces shall not, to an appreciable extent, refract doubly, crown glass is more desirable.

Different media vary greatly in density, and the denser a medium, the more it retards the velocity of light. Thus when a ray of light,

travelling in air, reaches the surface of a piece of glass, its velocity becomes instantly reduced by more than one-third, and after travelling through the glass at this lessened speed, it regains its former velocity as it emerges again into air.

A consequence of this remarkable property of media is, that whenever light enters a medium of different density, other than perpendicularly, it is bent or refracted from its former course. If the medium be homogeneous, the light pursues a perfectly straight course through it, and is only refracted at its surfaces.

To show how this refraction takes place, it will be necessary to touch lightly on some of the elementary properties of light, which, according to Clerk Maxwell's generally accepted theory, consists of electrical waves, accompanied by magnetic waves at right angles to them. The waves travel onwards, though the actual electrical and magnetic displacements are at right angles to the direction in which the waves travel: just as when a stone is flung into a pond, though the waves travel rapidly towards the banks of the pond, the actual particles of water do not move towards the banks at all, but only rise and fall, mounting to the summit of a wave, and then sinking into the trough left behind it, so that the motion of the particles is at right angles to the direction in which the waves are propagated.

The undulations which constitute light differ, of course, vastly from those of the water, in that it is not matter which moves, and also in that the undulations of light are not in one transverse direction only, but are in all directions transverse to the direction in which the waves are propagated. Moreover, while the waves in a pond travel along a surface only, light travels in all directions from its source. Therefore, when light is generated at a point it goes forth therefrom, if unhindered, not in ever-widening concentric *circles*, like the waves of the pond, but in ever-expanding *spheres*, which are called *wave-fronts*. The propagation of light is ever perpendicular to its wave-fronts, just as in the case of the waves of water.* It is thus that light travels in straight lines which were formerly called "rays," but which actually are nothing more than the lines of direction along which the disturbance proceeds, just as imaginary lines, drawn in all directions from the spot where the stone entered the water, will be radii of the circles, and represent the directions in which the waves advance. Now, if the pond

* The rectilinear propagation of light depends on the principle of interference. Every fresh point of disturbance would become the centre of expanding waves with spherical fronts, were it not for interference with similar waves from the neighbouring points of disturbance which, by composition, result in a general forward advance, as expressed by the law of Huygens, that the disturbance at any point of a wave-front is the resultant of the separate disturbances which the different portions of the same wave-front, in any one of its earlier positions, would have occasioned if acting singly.

be perfectly circular, the waves generated by a stone dropped exactly into its centre will break upon the shore with the direction of their wave-fronts unaltered. If, however, the pond is not circular, wherever the waves meet the shore obliquely, the direction of the wave-fronts will be altered as they break upon it. Similarly with light; when it falls *perpendicularly* on the surface of a new medium, the direction of its wave-front is unchanged, but if it fall *obliquely* the wave-front is altered in direction. There is this difference, however, that the light waves are not like those of the water, broken upon the shore, but go on travelling through the medium in a new direction—a direction perpendicular to the deflected *wave-front*.

It may be asked, "How can the surface of a new medium alter the direction of the wave-fronts that meet it obliquely?" It depends on the fact already mentioned, that light does not travel with equal velocity through different media, but that its velocity is retarded in proportion to the density of the medium. We may illustrate it by a column of soldiers, crossing a plain at "quick march." When they come to a river that lies across their path, they have their velocity retarded, progression through water not being so rapid as progression through air. If the river lie exactly athwart their path, so that all the men in the front rank reach the

bank and enter the water simultaneously, the column has the direction of its front unchanged. When light comes from a considerable distance, the wave-fronts have practically no curvature, so that the ranks of soldiers may be taken to represent successive wave-fronts.

If, as in *Fig.* 10, the same column of soldiers find a river crossing their path *obliquely*, one man in

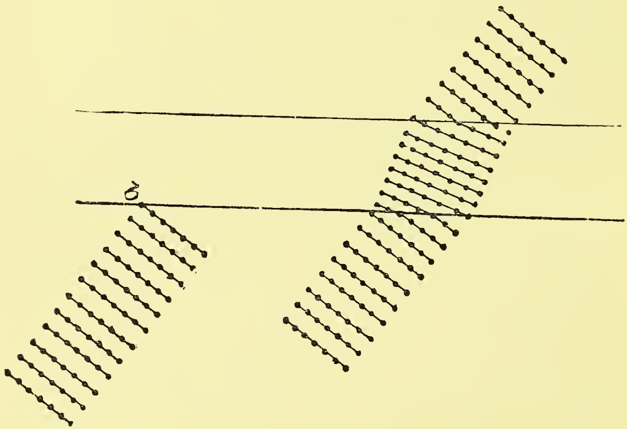

Fig. 10.—To illustrate refraction of light.

the front rank (*b*) reaches the water before the comrades by his side, and since he is retarded before they are, they gain on him, and the direction of the front of the column is altered. The same may be said of all the succeeding ranks, and it illustrates how the successive wave-fronts of light, as they break obliquely on the surface

of a denser medium, become altered in direction. The same man, however (*b*), who was first to enter the water will also be first to get out of it, and he will then gain on his comrades (who are still in it) as much as he lost before, so that the column resumes its original frontage, and pursues its original direction, being only displaced a little to one side, as shown by the figure.

Let us now suppose the river to be succeeded by a triangular lake, to represent a prism instead

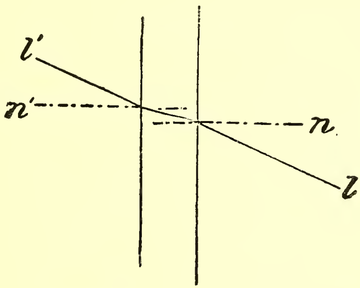

Fig. 11.—Refraction by a plate of glass.

of a plate of glass ; the original direction would not be resumed, for some would have a longer journey through the water than others. The man who first entered the water might have to leave it last, or, even if he left it first, he might not gain, on leaving it, as much as he lost on entering it. While the refracting surfaces of a pane of glass are parallel, as in *Fig.* 11, those of a prism are inclined to each other. The consequence is, that when rays of light traverse a prism, they

are bent from their previous course, and emerge in a new direction.

An imaginary line, perpendicular to the surface of a medium, is called a "normal" to that surface. Thus in *Fig.* 11, which represents a pane of glass traversed by a ray of light (l,l'), the dotted line n is the normal at the point of incidence of the ray, and the line n' is the normal at its point of emergence. It will be seen that on entering the denser medium, the ray is bent (or "refracted") *towards* the normal (n), while at its emergence, it is refracted *away* from the normal (n').

Deviating Angle.—The deviation of light on traversing a prism is the combined result of the two refractions which it has undergone at the two surfaces. It may be said that

(*a*) The denser the glass; (*b*) the greater the angle of the prism; and (*c*) the more obliquely it is traversed by the ray, the greater is the total deviation.

With prisms of ordinary crown glass, the "deviation" of a ray of light passing through the prism (in the direction of "minimum deviation," p. 26) is slightly more than half the edge-angle of the prism, if it be a weak one, and increasingly more than half as the edge-angle increases.

In *Fig.* 12 a ray (e) from the right hand side of the diagram is incident on the prism. The dotted

line (n) is the normal at its point of incidence, and i is called the angle of incidence. On entering the glass the ray (e) is bent towards the normal in the new medium, and on emerging from the prism it is bent away from the second normal (n'), so that its course is twice altered by its passage through the prism. The ray (e), on its incidence, was pursuing a course towards p, but, on its emergence, it proceeds towards f. The angle (d)

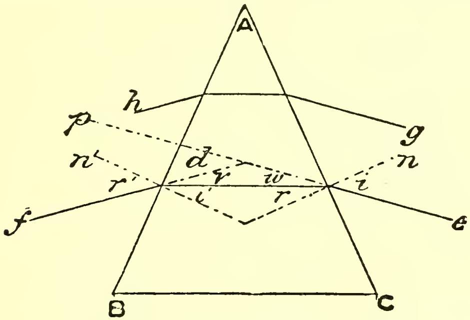

Fig. 12.—Refraction by a prism.

contained between the forward prolongation of the incident ray (e), and the backward prolongation of the emergent ray (f) is the *deviating angle*, or, shortly, the "*deviation*" of the prism. The amount of deviation suffered by a ray is the same, whatever part of the prism it traverses, provided it fall on the first surface with the same obliquity. Thus, rays of parallel incidence, as g and e, in *Fig.* 12, also emerge parallel as h and f. In the figure, the first surface-deviation is w, and the second surface-deviation

is *v*. It is evident at a glance that the total deviation by the prism is the sum of the two surface-deviations. The figure is so constructed that the angle of incidence (*i*) at the first surface, and the angle of emergence (*r*) at the second surface are equal, and therefore the two surface-deviations (*w*, *v*) are equal. When we speak of the deviating angle of a prism, we assume the

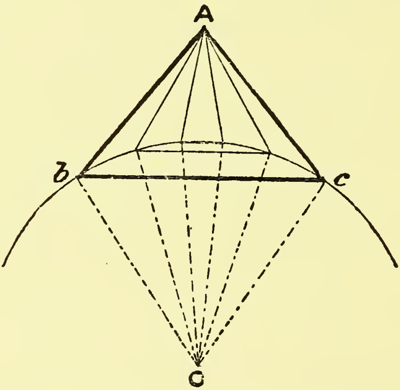

Fig. 13.—To show the proportion between the refracting angles (*A*), and the deviating angles (*O*) of prisms.

deviation to be that of a ray subject to these conditions, for, as we shall see, under other conditions the light-deviation is greater. With prisms of ordinary crown glass, the minimum deviation of light is slightly more than half the angle between the two faces ("the refracting angle," as it is called) of the prism, if it be a weak prism; but as prisms become stronger, the greater

is the excess of the prism-deviation over the half of the refracting angle.

In *Fig.* 13 I have shown in an exact though simple way, how the deviating angles of prisms increase more rapidly than their refracting angles. Three prisms are delineated, in principal section (*vide* p. 5 and *Fig.* 3), with their apices meeting at *A*. The dotted lines from the extremities of the base of each prism enclose between them the deviating angle of that prism. A prism only a trifle stronger than the largest in the figure would have a deviating angle *equal* to its refracting angle. But since for clinical purposes we never approach anything like this strength, and the smallest prism in the figure represents about the largest we prescribe, we have never to do with a deviating angle much more than half the refracting angle.

The same figure shows us how easily we may find the deviating angle of any prism by construction. Draw any circle about a point (*O*), and take any point (*A*) outside the circle, such that the distance *A O* shall be to the radius of the circle in the ratio of the refractive index of the glass. Thus, with crown glass, since its average refractive index is 1·54, if we make the radius one decimetre we shall make the distance $A\ O = 1{\cdot}54$ decimetres. Then with *A* as apex, draw the prism so that its base (*b c*) shall be a chord of the circle, and its refracting angle

(A) shall be bisected by a straight line between A and O. Then lines drawn from the centre (O) to the extremities of the base enclose the deviating angle required.

With weak prisms there will be only a small error if we express the ratio of the deviating angle to the angle of refraction by the decimal figures in the index of refraction of the glass. Thus, if the glass have an index of 1·5, the deviating angle is half (·5) of the refracting angle. If the index be the usual one of 1·54, the deviating angle is rather more than half (·54), and so on. For exact formulæ, the reader is referred to the Appendix, to which the mathematical treatment of prisms is relegated. Dr. Edward Jackson affirms that of late years the tendency has been to raise the index of crown glass, and that many years ago it was nearer 1·52. His proposal before the Ninth International Congress, to have all prisms marked according to their deviation of light, instead of by the angle between their faces, has been generally approved. The Committee appointed to consider the proposal recommended, in 1888, that:—

"(1) Prisms ought to be designated by the number of degrees of 'minimum deviation' they produce.

(2) Where intervals of less than one degree are desired, half degrees and quarter degrees should be used.

(3) To indicate that degrees of deviation are meant, the letter *d* should be used; thus, prism 2°*d* will indicate a prism that produces a minimum of two degrees."

In the following table I have shown the angle of deviation for prisms having edge-angles of from 1° to 12°, if the refractive index be taken as 1·54. It will be seen that even with the highest of the series there is an error of only

TABLE I

Angle Between Faces	Angle of Deviation	Angle Between Faces	Angle of Deviation
1°	32′	7°	3° 48′
2°	1° 5′	8°	4° 20′
3°	1° 38′	9°	4° 54′
4°	2° 10′	10°	5° 26′
5°	2° 42′	11°	5° 59′
6°	3° 15′	12°	6° 32′

half a degree in assuming, as we are wont to do, the deviating angle to be half the edge-angle.

It must be confessed that the dioptric measurement of prisms lays a burden on the workman, for whereas, formerly, he could test the correctness of the edge-angle at all stages of his work, it is impossible to test the deviation of a beam of light until the polishing process has begun. Perhaps the best plan is that recommended by Prentice—to keep a large number of prisms in stock, of all sizes, and select the one whose deviation is nearest the prescription.

In attempting to make a prism of definite dioptric power, the workman should have a good idea to start with of the angle which, when imparted to the glass, will be most likely to satisfy the requirements.

The simplest way of doing this without calculation*, and yet with perfect accuracy, is to employ the author's suggestion of working with an ordinary semicircular protractor from without, as in *Fig.* 14. The workman should place the pivot of the dividers at a point whose distance

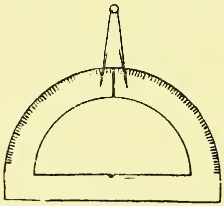

Fig. 14.—To show how a protractor could be used to make prisms of definite deviations.

from the centre of the arc is 1·54 times greater than the radius of the arc, as shown, greatly reduced, in the figure. The legs of the compasses should be opened symmetrically with respect to a line joining their pivot and the centre of the arc.

* Anyone wishing to do this by *calculation* may multiply the prescribed angle by two, and subtract nine minutes for each degree in the prescribed angle if the prism is a weak one, or ten minutes for each degree if the prism is a strong one (over 6°*d*). Thus to produce a prism of 2°*d*, he would cut the glass to an angle of 3° 42′; that is, to 2 × 2° *minus* 9 × 2. To make a prism of 9°*d*, he would cut the glass to twice nine degrees, *minus* ten times nine minutes, or 16° 30′.

On the same principle, a special instrument could be made, as in *Fig.* 15, by cutting out a piece of sheet celluloid to the shape there figured, making the distance of the point *c* from the centre (*O*) of the arc $3\frac{1}{4}$ inches, and cutting the arc along the edge of an ordinary graduated semicircle of 6-inch radius. The graduations of

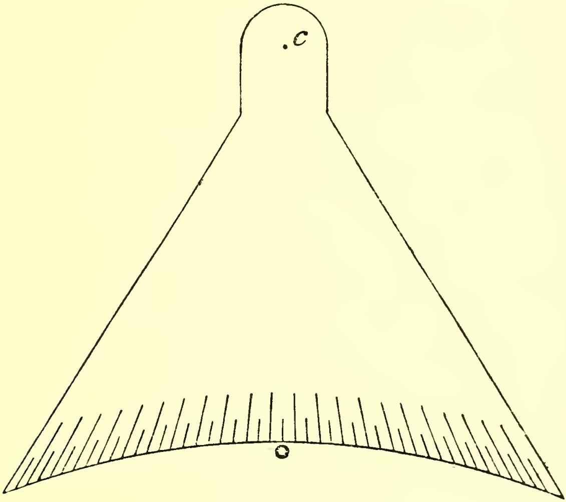

Fig. 15.—Author's deviation calculator. The distance of *c* from the centre of the circle is 1·54 times the radius of the latter. The arc is marked in ordinary degrees.

the semicircle transferred to the edge of the arc indicate the points for the graduation of the latter. The short lines, however, from those points should, as represented in the figure, be drawn so as to point towards *c*.

The advantage of this little appliance is that the measurements being made once for all, a pair of dividers need only be placed with their pivot on *c*, and their legs be opened wide enough to enclose the required number of degrees for the deviation of light (taking care that the two legs are kept equidistant from *O*). The angle between the legs of the dividers is the edge-angle of the required prism. If a short arm were attached to the protractor of *Fig.* 14, with a spot on it to indicate the position of the pivot of the compasses, that spot being surrounded by two or three concentric circles, it would be the same thing in principle, and answer practically almost as well.

Minimum Deviation.—When the points of incidence and emergence of a given ray are equidistant from the edge of the prism, the ray is said to traverse the prism *symmetrically*, because the angles of incidence and emergence are then, as shown in *Fig.* 12, equal. A ray under these conditions suffers less total deviation than if it passed through the prism in any other direction, and is therefore said to traverse the prism in the "direction of minimum deviation," or, to put it another way, the *prism* is said to be (in reference to the ray) in its "position of minimum deviation."

This is easily demonstrated by looking at a small object through a prism, and then rotating

the prism about its axis,* when the apparent displacement of the object will increase with each increment of obliquity imparted to the prism. It will be found that there is a certain position which gives smaller apparent displacement than any other, and that when the prism is in this position small departures from it have little appreciable effect on the amount of deviation. The position of minimum deviation is thus most suitable for measuring the optical strength of prisms, because it is the position in which slight accidental aberrations have least effect.

The fact that a ray of light is more strongly deviated when it passes through a prism in a more oblique direction, is an important one in optics, accounting as it does for "prismatic astigmation," and for one or two varieties of the distortions produced by strong prisms. For these the reader is referred to Chapter XIV.

Fig. 14 only shows how to find the minimum deviation of prisms. It remains to describe a graphic method of finding their deviation of light when it does not traverse them symmetrically, when, in other words, the angle of emergence differs in size from the angle of incidence. A simple mechanical method the author has adapted for this purpose, after a well-known theorem, is shown in *Fig.* 16.

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* The axis of a prism is an imaginary straight line parallel to the edge, and midway between the edge and the base.

Let two circles be described about a common centre (O), with their radii in the ratio of the refractive index. That is, if the radius of the smaller circle be 1, that of the larger will be 1·54 in the case of crown glass. At zero point (I) of the smaller circle insert a pin perpendicularly to the surface, and after cutting a triangular piece of paper (a b c) to represent any given prism, lay it down with

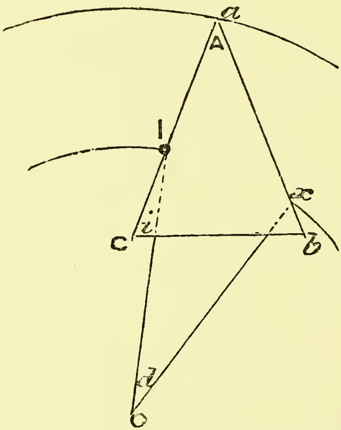

Fig. 16.—Mechanical method of showing how much the deviation produced by a prism alters with the angle of incidence. The smaller arc should be graduated from *I*.

its side (a c) touching the pin (I), and its apex (a) touching the outer circle. The right hand edge (a b) of the movable triangle will then mark off on the smaller circle the arc of the angle of deviation, while the angle (i), between the margin (a c) of the triangle and the line (I o) from the pin to the centre of the circle, represents the angle of incidence for each position.

By rotating the triangle into various positions, ever keeping the apex in contact with the outer circle, and the left-hand margin in contact with the pin, we can find in a moment the arc of the deviation produced for any required angle of incidence; that is, for any value of i the deviation is represented by d. It will be noticed that the position of the paper triangle in which it marks off least of the smaller circle, and therefore indicates the smallest possible angle of deviation, is when the line which bisects its angle (A) passes through the centre (O). This is the "position of minimum deviation." It may be said, in passing, that the size of the angle of emergence is represented in every position by the angle $b\ x\ o$.

It is sometimes desired to find the deviation suffered by a ray which enters one surface of a prism perpendicularly, and which, therefore, is only refracted at the second surface. Indeed, Prentice recommends this position for the measurement of prisms, as being most suitable for his "Prismometer." To find what is the deviation to be expected under these circumstances, we have only to revolve the paper prism till the margin ($a\ c$) coincides with the radius ($I\ o$,) when the other margin ($a\ b$) will mark off the arc of the deviation required.

Dispersion.—By dispersion, or "chromatic aberration," is meant the breaking up of light into its constituent colours. It is a property possessed in greater or less degree by all prisms, since the differently coloured lights, being unequally refrangible, are deflected through different angles, the violet light experiencing

the greatest deflection and the red the least, while the paths of the lights of intermediate colours are spread out between these two extremes. The whole solar spectrum can thus be received upon a white screen arranged to intercept a pencil of light which has traversed a strong prism. This property of dispersion, so valuable in spectrum analysis, is a serious disadvantage in ophthalmological prisms, from the coloured margins it appears to give to objects, in proportion to the strength of the prism used.

It is only fairly weak prisms, therefore, that can be clinically employed with much advantage. It is not often advisable to exceed $3°d$ or $4°d$, though the limits depend on the visual acuity, and habits of observation, of the patient. In amblyopia from corneal nebulæ, stronger prisms can be borne. The dispersive power of crown glass is so much less than of flint glass that it is invariably employed for ophthalmic purposes. Achromatic prisms can be made by cementing together two prisms, one of crown, the other of flint glass, with the apex of one to the base of the other, since the ratio between the refractive and dispersive power is different in the two kinds of glass.* But though suggested long ago, such combinations have been precluded by their weight from clinical use.

* The dispersive power of crown glass is to that of flint glass as 33 to 52; whereas the refractive power is about as 31 to 32, though it varies greatly with different specimens, and must be determined for each.

CHAPTER III

DIAPRISMETRY

It is certainly of advantage for the surgeon to possess a means of noting the optical strength of the prisms he meets with or uses. This is especially the case in scientific enquiries.

Since instruments for the measurement of the edge-angles of prisms have long been known by the name of "goniometers," we may choose the name of "diaprismeters" for such as measure their optical power.

The simplest form of goniometer is the "prism-measure" of the Geneva Optical Company, but this acquaints us only with the geometrical—not with the optical—properties of prisms; and what the surgeon needs alone to know is not their refracting angle, but their deviating power over a beam of light. It has been noticed already that this "angle of deviation" is in each case about half the angle of the faces—with weak prisms inconsiderably more, and with strong ones increasingly more, than half; but apart from this difference, the refractive index of different specimens of crown glass varies a little, so that Donders* truly observed of the angles of

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* *Accommodation and Refraction of the Eye* (Sydenham Society, p. 132).

deviation, "If we wish to know these accurately, it is necessary to determine the deviations for each glass separately." To do this with certainty and ease, two simple methods were devised in 1886, one most suitable for practice, the other for demonstration purposes. The first can be made without expense, and employed without artificial light. It consists simply of a strip of paper or cardboard suspended horizontally on the wall, at such a height from the floor

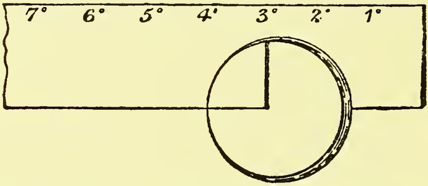

Fig. 17.—Author's first mode of diaprismetry.

as to be about level with the eyes of the observer. The card itself is marked, as in *Fig.* 17, in tangents of degrees* for a certain distance. All that is necessary to measure deviating power is to hold the prism at this distance, with its apex to the left, and so that its upper border appears just beneath the line of degrees, as in the figure. The vertical border of the card appears necessarily displaced towards the edge of the prism, and

* A scale of this kind has been used since May, 1886, by Curry & Paxton, for whom the author designed it, to measure the deviating angles of their prisms. In later years several slight modifications of this scale have been made by others while still retaining its principle.

points upwards to the number which expresses in degrees, at once and without calculation, the required deflecting angle of the prism. In the figure this is seen to be 3°. One precaution alone needs to be observed: if the lower edge of the cardboard seen through the prism appears disjointed from the level of the rest, as in *Fig.* 18, the apex of the prism is not pointing exactly to the left, but is either too high or too low, according as that part of the broken line seen through the prism is above or below the remainder. A

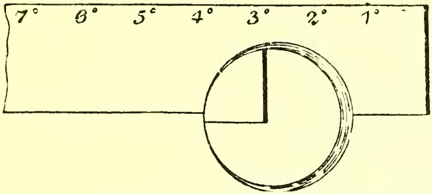

Fig. 18.—A prism not held truly.

slight rotation is therefore necessary to correct this, till the lower border of the cardboard appears at one level, as in *Fig.* 17.

The card can be graduated for any distance at which it is found most convenient to hold the prism. The distance of the observer's eye from the prism is of no consequence. The accompanying table makes it easy to graduate a strip of paper or cardboard. It gives the distances from the right-hand border of the card at which the several degrees are to be marked

upon it, assuming the prism to be held at a uniform distance of six feet (first column), or two metres (second column).

TABLE II

For marking a card in tangents of degrees at 6 ft. (column A); or 2 metres (column B)					
	A	B		A	B
1°	1·25 in.	3·49 cm.	9°	11·4 in.	31·68 cm.
2°	2·5 ,,	6·98 ,,	10°	12·6 ,,	35·26 ,,
3°	3·7 ,,	10·48 ,,	11°	14·0 ,,	38·88 ,,
4°	5·0 ,,	13·98 ,,	12°	15·3 ,,	42·5 ,,
5°	6·3 ,,	17·5 ,,	13°	16·6 ,,	46·17 ,,
6°	7·57 ,,	21·0 ,,	14°	17·9 ,,	49·86 ,,
7°	8·84 ,,	24·56 ,,	15°	19·3 ,,	53·59 ,,
8°	10·12 ,,	28·11 ,,	16°	20·64 ,,	57·35 ,,

For a different range, the measurements would of course differ in proportion: thus, should it be decided always to hold the prism at three feet, the distances would be half as great as in the table; if at twelve feet, twice as great.

If the card be graduated in centimetres and used at the distance of a metre it measures the strength of prisms in prism-dioptres; but the author of that unit, Mr. Prentice, rightly prefers to graduate a card on the same principle with six-centimetre intervals for use at six metres, calling it a prismometric scale, and the mode of measurement therewith is the same as that of the earlier scale in *Fig.* 17.

The second method of diaprismetry is only intended for *demonstration* of the deviating angle

of prisms, but needs a more concentrated illumination than diffused daylight. It consists of a strip of wood, as in *Fig.* 19, laid flat on a table, with a shorter vertical strip (*b*) fixed at right angles to its further end. Upon the longer strip a cylindrical lens (*c*) is fixed, with its axis horizontal, so as to throw a line of light from a bright point upon the strip (*b*), which is marked in tangents of degrees; so that a prism held before the cylinder (*c*), with its base-apex line vertical, at once makes the bright line move to that degree which

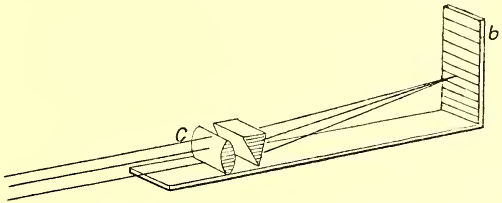

Fig. 19.—Rough mode of diaprismetry.

indicates the deviating angle. The lens can be spherical if illuminated from a bright slit, and in any case it is well, if the light is poor, to combine with the cylindrical lens a spherical one of such strength that the source of light shall be at its principal focus.

Unit of Diaprismetry.—Three proposals are in the field as to the unit of measurement for the deviation of prisms—the degree, the centrad, and the prism-dioptre. Of these the centrad has been selected by the American Ophthalmological Society, though in this country the degree is still preferred.

The *degree* is a familiar unit, and if "possession is nine points of the law" should hold its own, unless some really surpassing merit attaches to its rivals. We measure every other ophthalmic angle in degrees, and it would certainly seem strange to measure a deviation of the eyes in one unit, and correct that deviation in another. Perhaps when some other unit has established its superiority sufficiently to replace the degree for hyperphoria it will be time to accept it as an exclusive unit for prisms.

The fact that the earth completes its course round the sun in a little over 360 days no doubt gave rise to the division of the circle into 360 units. The odd days can be neglected, since the orbit, owing to its ellipticity, is not a perfect circle.

It is unlikely that a unit based on such tremendous and regular phenomena, and withal, one which lends itself so readily to sub-division, should lose its hold upon the scientific mind, for, after all, astronomy is the queen of sciences. Is it wise to attempt to displace that which has so stable a footing? *

The "*Centrad*" was proposed by Dr. Dennett, of New York, who claims for it three advantages over the degree as follows—

1.—Being a new unit proposed solely for the deviations of prisms, it might lessen the possibility of confusing the two methods of marking prisms.

2.—It is a smaller unit than the degree. A centrad is slightly more than half (·57) of a degree.

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* In saying this the author may possibly be too much biassed by the conservative atmosphere of an old country. After all, however, America is sent round the same kind sun!

3.—It has a certain relation to the metre-angle, in that half the number of centimetres between the pupils indicates approximately the number of centrads in one metre-angle.

The centrad is taken from the well-known "circular measure." It is the hundredth part of a "radian," a radian being the angle subtended at the centre of a circle by an arc which is equal in length to the radius. Its ensign is an inverted delta. Circular measure has a certain theoretical interest.

While the author is inclined, it will be seen, to deprecate the invention of special units for tiny departments of science, wishing rather to preserve the breadth and unity of the whole, it must be confessed that a departure or gradient of *one in a hundred* has much to recommend it. Next to our old and well-tried friend, the degree, probably no better unit could be chosen.

The unit proposed by Mr. C. F. Prentice is the "*Prism-Dioptre*." It has much to recommend it, and its publication by him preceded and no doubt suggested the centrad. Its ensign is an erect delta. The size of a single prism-dioptre differs inappreciably from that of a centrad, both being the departure of 1 in 100, that is, a deviation of one centimetre at one metre distance, but the centrad is measured on the arc, and the prism-dioptre on the tangent. The difference between the two units is shown diagrammatically in *Fig.* 20. In popular language, one may be said to be measured on the circle and the other on the square. The relation to the metre-angle of the P.D. is practically the same

as the centrad, though not quite so perfect.* Its chief recommendation is the ease of measuring on the straight, and also its beautiful relation to lenses, and their decentring. It has the disadvantage, however, of not being theoretically suitable in high numbers for multiplication : for instance, two prism-dioptres form an angle less than twice one prism-dioptre. A prism-dioptre is 34′ 35″.

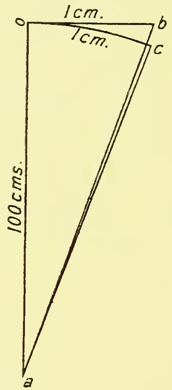

Fig. 20—To show, with enormous exaggeration of each, the difference between a prism-dioptre *o a b*, and a centrad *o a c*.

The *measurement* of deviations is as easy with one unit as with another, and the scale figured on p. 32 has been adapted by Prentice to the prism-dioptre, by

* The metre-angle is the angle whose *sine* is half the inter-ocular distance in terms of a metre. Since the centrad is an arc measurement, and the prism-dioptre is a tangent measurement, the former approaches the sine more nearly than the latter. Arcs lie between sines and tangents. (*See* Appendix, p. 192.)

having it marked in 6 cm. intervals instead of in degrees, while a beautiful instrument has been constructed by him called the "Prismometer," acting on the same principle, to use where great refinement is called for.

To measure in centrads, a short arc would have to be used, marked in hundredths of its radius, or a tangent scale be made in tangent-centrads.

Mr. Prentice has, with characteristic ability, shown how the adoption of the prism-dioptre would reduce the decentring of lenses to extreme simplicity.

A lens of 1D displaced laterally one centimetre has the effect of a 1 P.D. prism; displaced 2 cm. of 2 P.D., and so on. A lens of 2D similarly displaced, has the effect of a 2 P.D. prism.

We should have only to remember that the prismatic effect is that of the decentring in centimetres, multiplied by the dioptric strength of the lens.

These advantages are for all practical purposes shared, as we shall see, by the centrad, which the author prefers to call an arc-centune, and Mr. Prentice's unit a tangent-centune—names which would bring to mind always, what is often forgotten, namely the practical equality of the two units.

All units based on the metric system lose for the present much of their practical value from the fact that, at least in this country, the workmen in manufacturing houses still make all their measurements in the usual fractions of an inch.

This is true, not only of all the dimensions of spectacle frames, but also of the decentration of lenses. It is a fact which we cannot afford to overlook, that in spite of the perpetual prescriptions given

by oculists in millimetres, British workmen at once convert these into their favourite "eighths" and "sixteenths" before setting to work, reckoning, I am told, three millimetres to an eighth.

This being so, the proposed metric substitutes for the degree cease at present to possess much special advantage, for ophthalmic surgeons are inclined to prescribe at once in the unit which the workman will use, and thus avoid a possible source of error in his translation of one unit into the other.

In America, prisms are, it is believed, generally prescribed in centrads, and manufactured in prism-dioptres.

The former practice is due to the adoption by the American Opthalmological Society of the centrad, and the latter to the greater ease of measuring on the straight. Escape from the dilemma is proposed as follows :—

Since there is nothing in the new name of centune to indicate whether it should be measured on the arc or tangent, the course pursued in the author's mind is to give it the generic significance of a one-per-cent deflection, leaving its measurement to be made by arc or tangent as the case may require.

For exact calculation, or when high multiples are employed, it might be judicious to specify arc-centunes or tangent-centunes ; but for ordinary clinical prisms, and decentring, the difference may be considered negligible in practice, and that for two good reasons :—

1.—Though the focal surface of an ordinary lens is assumed by authors, for convenience, to be a focal *plane*, it is in reality a hollowed surface.

It is only by confining ourselves to very small departures from the principal axis that we are justified in considering the curvature negligible. But decentring involves considerable departure from this axis. If, therefore, we assume this difference negligible, we may *clinically* reckon that between the arc and the tangent measurement negligible.

2.—The effect of *spherical aberration* has been overlooked hitherto in all decentration problems. In virtue of it a lens decentred one centimetre produces a greater deviation in prism-dioptres than the number of dioptres of the lens. Thus a 1D lens, decentred one centimetre, produces slightly more than a prism-dioptre of deviation, somewhat nearer therefore a centrad.

For such reasons, while still giving Mr. Prentice the credit undoubtedly due to him of being the first to apply the metre unit idea to prisms, the two competitive units might, in the author's view, be with advantage merged into one, under the common name of centune, qualifying it further when need requires, as arc-centune or tangent-centune. To give an example, a prism of two centunes, or in other words a two-per-cent prism, would deflect light by two centimetres per metre. It could be prescribed clinically as a 2 % prism, or be described for accurate calculation as a 2% (arc) prism, or as a 2% (tan) prism.

The advantage of the name centune lies in its flexibility, since it only means a one-per-cent unit, which can be differentiated as required. The name centrad on the contrary forbids tangent measurement, while prism-dioptre is too narrow a title for a unit suited for usefulness in other departments of science.

CHAPTER IV

TESTING AND ADJUSTING THE APEX

Circular prisms, as usually sold, have their apex and base indicated by a slight scratch from a diamond. On testing a series of prisms by a simple plan, to be described immediately, I noted inaccuracy in a large proportion. An obvious consequence of inaccurate marking is that prisms may not be set quite truly in the trial-frame; yet any departure from precision in this particular alters an experiment. For example,

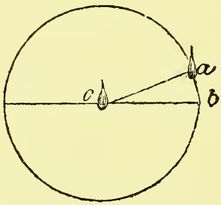

Fig. 21.—Showing the effect of setting a prism by a wrong mark. The marked apex is towards *b*, and the true apex is towards *a*, so that the flame *o*, which should appear at *b*, appears at *a*.

if it be desired to see whether a patient can overcome a prism of 15°*d*, with its apex exactly in or out, an aberration of the base-apex line through only a ninetieth part of a circle has the effect of not only lessening in a trifling degree the desired horizontal deflection which it is the aim

of the prism to produce, but also of introducing an unsuspected vertical deflection of more than 1°. This is shown in *Fig.* 21, where the actual flame (*o*) appears displaced to *a* instead of to *b* ;—the prism's base-apex line being *o a*, though marked as if it were *o b* ; hence *a o b* is the angle of error in setting the prism. At six metres distance the flame would appear more than eleven centimetres higher than it really is. Now, it is well known that vertical diplopia greatly embarrasses the neuromotor apparatus, and in some persons it cannot be overcome at all when greater than one degree, so the prism in the experiment may be laid aside with the verdict that it is too strong to be overcome, when really it is the concomitant accidental *vertical* diplopia created by it which cannot be overcome, or, at least, the effect of which alters the result.

If anyone will take the trouble to test the marks upon his own prisms, he will find how frequently they are misplaced ; the test can be easily made by the following simple device. Draw upon a sheet of paper two fine straight lines intersecting each other exactly at right angles, as in *Fig.* 22, and hold the prism horizontally above them, so that on closing one eye the supposed apex apparently coincides with the point of intersection of the lines, and the prism appears to touch the horizontal line only at that point, and does not in the least overlap it on either side. If the

prism is correctly marked, the vertical line appears unbroken, as in *Fig.* 22, but if not, the part of the line seen through the prism appears disjointed from the rest and displaced, as in *Fig.* 23, towards the side of the real apex. It only remains to rotate the prism till the unbroken appearance of *Fig.* 22 is gained; then the point on the prism's edge in apparent coincidence with

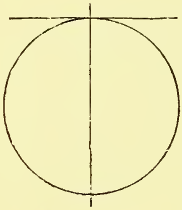

Fig. 22.—Mode described for testing the apex of a prism.

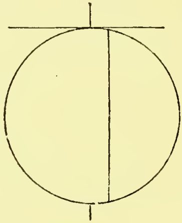

Fig. 23.—Appearance with a badly marked prism.

the point of intersection of the lines may be regarded with confidence as the real apex.

Greater distance of the prism from the vertical line magnifies the phenomenon, and makes the test more delicate. The true base of a prism can be found and marked in just the same way as the apex. To those who manufacture prisms in large quantities it may be permissible to suggest a special apparatus which I have designed to mark the apex with mathematical accuracy. Each prism should be laid on a horizontal glass plate, with one longitudinal line at some little distance beneath it. Two pins or projections

from the upper surface of the glass plate, half an inch apart, and on a line strictly at right angles to the line beneath, should be provided to press the prism against, while it is rotated till the longitudinal line appears continuous, then a diamond held in a sliding arm, and working midway between the two pins, would scratch the apex.

Adjustment of Prisms in the Trial-frame.—If the apex and base of a circular prism are

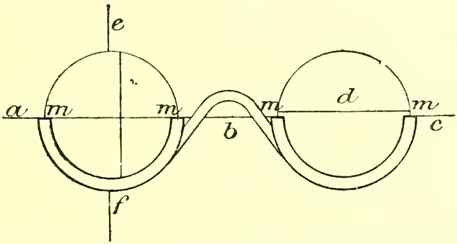

Fig. 24.—Two prisms, one vertical, the other horizontal, incorrectly set in a trial frame.

correctly marked, there is no difficulty in setting it with the base-apex line either exactly vertical or horizontal. But should there be any doubt about their precise position, the following simple expedient will at once enable us to detect and correct any malposition.

If the base-apex line is wished to be horizontal, as with the right-hand prism of *Fig.* 24, hold the trial-frame a few inches from the eyes, so that, as represented in the figure, the upper extremities (*m m m m*) of its sockets coincide, to all appearance,

with a horizontal line (*a b c*) upon the wall, door, or window: if this line appear continuous, the prism is correctly set, but if that part of it visible through the prism appear disjointed from the rest, as *d*, the apex is either too high or too low, according as the apparent disjointment is upwards or downwards. In the figure the apex is too high, and the correction is therefore easily made by rotating the prism so as to depress the apex till the line appears continuous.

If the base-apex line is wished to be vertical, as with the other prism of the same figure, the horizontal line on the wall must be supplemented by another at right angles to it, as *e f* in the figure: again adjusting the frame as before, to the horizontal line of sight, the rectitude of the prism is now indicated by the appearance of the *vertical* line. If its course appears undeviating, the setting is faultless, but if, as in the figure, the part seen through the prism is dissociated from the rest, the apex is shown to be too much to the same side, and it therefore needs readjustment.

With artificial light, the author has shown how images by double internal reflection can be utilized to advantage, as described in the next chapter. The best way to use them is to place the trial-frame on one's own face, and rotate the prism till the faint image appears level with the flame, or vertically above it.

CHAPTER V

IMAGES BY INTERNAL REFLECTION

HITHERTO we have only considered the *refraction* of light by prisms, but at each surface a smaller portion is also reflected. The more oblique the incidence of a beam of light, the greater is the

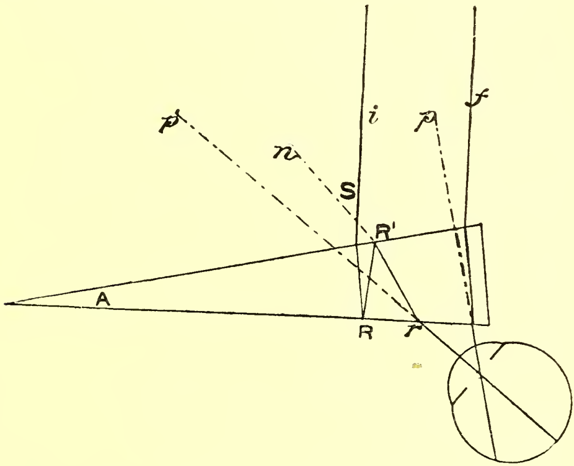

Fig. 25.—Image produced by internal reflection within a prism.

proportion reflected, and the brighter is the image produced by reflection.

The light reflected from the first surface of a prism is lost to the eye altogether, being returned

into space, but that which, after refraction at the first surface, undergoes internal reflection at the second surface, as at *R* in *Fig.* 25, is again reflected internally from the first surface at *R′* (though at each surface a large proportion escapes) before it finally reaches the eye by refraction at *r*. Even at *r* a portion is again reflected internally, and this internal reflection from surface to surface goes on till the base of the prism is reached, or till all the light is, for practical purposes, lost by escaping from the prism. With a weak prism we may, on looking through it at a flame, see a series of images from internal reflection, especially if we hold it obliquely to increase their brightness, but each becomes fainter than the last. They, and the flame, all appear in one straight line, which is parallel with the base-apex line of the prism. Only the first of these images is illustrated in *Fig.* 25.

The following is a brief study of what takes place. When the incident ray (*i*) traverses the prism in the direction of minimum deviation, the angle of reflection at *R* is equal to the refracting angle (*A*) of the prism, and each subsequent reflection is by an angle greater than the last by twice the angle of the prism (3A, 5A, 7A, etc.). It is under these conditions that the course of the rays can be studied to most advantage, for one finds by experiment that moderately tilting a prism away from this position has very little effect on the apparent position of the image.

Since the ray which escapes at *r* into the eye determines the direction in which the faint image is projected, the projection is indicated by the dotted line *p′*. The ray *f* is one which enters the eye by refraction only, and is projected in the direction of the dotted line *p*. The angle between the two dotted lines *p* and *p′* represents, therefore, the visual angle between the bright image of the flame seen by refraction only, and the first faint image seen by reflection as well. Since the imaginary line joining these two images is exactly parallel to the base-apex line, we are provided with a very beautiful way of setting a prism exactly horizontal or vertical: we have only to see that these two images are horizontal or vertical relatively to each other. In other words, the faint image of a flame viewed through the centre of a prism points exactly to the apex of the prism, and it is curious that so simple a guide does not appear to have been used earlier.

Could we not utilize also the *distance* between the bright image of a flame and its second image for measuring the strength of a prism? For the stronger the prism the greater their distance. Nothing is easier than to say what figure of a scale is covered by the second image of a flame placed at the zero of the scale and looked at through a prism, and were the refractive index of glass a constant quantity, this plan would provide us with a more delicate test of the strength of a prism than any in use. But

unfortunately there are slight differences in the refractive index, and the visual angle depends not only on the angles of reflection, but also on those of refraction. The formula I have made, however, shows that the figure on a tangent scale pointed out by the second image, indicates, without much error, that the angle of the prism is nearly one-third, *and the deviating angle nearly one-sixth*, of that expressed by the scale. If we know the index of refraction, or have such confidence in the uniformity of the glass as to take its average for granted, we can find the exact strength of the prism by a formula. The distance of the scale should not be less than a metre, preferably two or five, if great delicacy be required.

Some may be interested to know how to make the calculation. Let a line (n) be drawn from R', such that if it represented a ray incident at R' its refracted ray would be $R'r$. Then the angle (S) between this line and the ray i is the angle expressed by the figure on the scale covered by the second image, for it is evident that light proceeding from n, and light proceeding from i enter the eye as one in the beam from r. But the ray $R'r$ forms an angle of $\frac{3}{2}A$ with the normal, therefore the angle of incidence of the ray n is $sin.^{-1}(\mu\ sin.\ \frac{3}{2}A)$. This gives us the inclination of n to the normal. The inclination of i to the normal is evidently $sin.^{-1}(\mu\ sin.\ \frac{A}{2})$; but these two angles together compose S. So that $S = sin.^{-1}(\mu\ sin.\ \frac{3}{2}A) + sin.^{-1}(\mu\ sin.\ \frac{A}{2})$. From this a table could be easily constructed for any given index of refraction.

In that which follows the index of refraction is taken as 1·54.

TABLE III

Refracting Angle of Prism	Projection of Faint Image on a scale	
1°	3°	4′
2°	6°	9′
3°	9°	14′
4°	12°	21′
5°	15°	27′
6°	18°	34′

It is rather strange that these images by internal reflection do not appear to have been employed for estimating the refractive index of glass. After finding the edge-angle by a reflecting goniometer, the refractive index of any ophthalmological prism could be determined, by their means, to a very high degree of accuracy from the formula given above.

CHAPTER VI

CLINICAL PROPERTIES

The ophthalmological use of prisms is to divert light so as to make it enter the eye in a different direction. Prisms, on this account, always produce an optical illusion, making objects appear in a different position from that which they actually occupy.

We have seen that when prisms are circular, their thinnest point is called the "apex," and their thickest part the "base," while an imaginary line between the apex and base may be called the "*base-apex line.*" With oval prisms there is also a true base-apex line discoverable by looking at a thin line through the prism and rotating the latter until the line appears unbroken, and passing at the same time through the centre of the glass. Discovery of the apex by double internal reflection is, with oval prisms, just as accurate as with any other shape. Objects viewed through any prism appear displaced in a direction strictly parallel to the base-apex line, and always *towards the edge.* It is customary to say that objects appear displaced towards the *apex*, and in practice there is no harm in using this convenient

expression, if not misunderstood. The design of *Fig.* 26 is to illustrate the difference, and show that the object (*o*) does not appear displaced in the direction (*d*) towards the apex (*a*), but in the direction (*e*) towards the edge (*b c*); for while *o* represents the real position of the object, *e* represents its apparent position as seen through the prism. If the prism be rotated, its edge, of course, rotates with it, but in all positions the

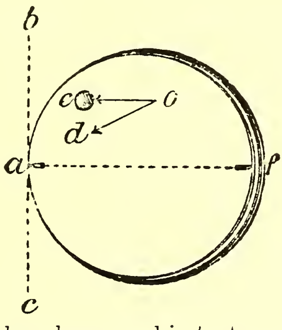

Fig. 26.—To show how an object at *o* appears displaced, not in the direction *d* towards the apex (*a*), but in the direction *e* towards the edge (*b c*), and parallel to the "base-apex line" (*a f*).

object will appear displaced to the same extent towards the edge, so that, in making a complete revolution of a prism, the false image will appear to revolve at an equal rate round the real position of the object.

Lines viewed through a Prism.—If instead of using a small object, we look at a vertical line on the wall, or the edge of a door, as *x x* in *Fig.* 27,

through a prism held several inches from the eye with its apex at *a*, we observe that the part of the line seen through the prism appears disjointed from the rest in a direction towards the edge of the prism, but the disjointed portion still appears a nearly straight line. If another line, such as *y y*, be in view through the prism at the same time, its disjointed portion will appear displaced in the same direction, and to exactly the same amount as that of the other line (*x x*).

Prisms compared with Spherical and Cylindrical Lenses.—It will be noticed that movement of the

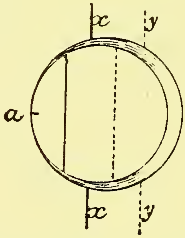

Fig. 27.—To show the apparent displacement of lines through a prism.

prism from side to side will not affect, in the slightest degree, the apparent position of the disjointed portions of the lines looked at through it. Herein a prism widely differs from a lens; every lateral movement of which causes apparent displacement of the object looked at, either with it if concave, or against it if convex.

Since the refracting surfaces of any prism are of necessity inclined to each other by the same

angle in every part of their area, it follows that the apparent displacement of an object is the same through whatever part of the prism it is viewed. It is true that rotation of the prism will alter the apparent position of the disjointed portions of the lines, but they will still continue (if the prism be weak) vertical. Herein a prism differs again from a spherical lens, rotation of which has no apparent effect whatever. A cylindrical lens differs also from a prism, in that when it is rotated, a line viewed through it appears distorted from the vertical.

When a prism is rotated, the apparent displacement of the disjointed portion varies in amount according to the rotation. The maximum displacement is obtained when the base-apex line is at right angles to the line looked at, and the displacement becomes less and less as this position is rotated from till the base-apex line becomes parallel with the line viewed, in which position the displacement is *nil*, or, rather, it appears *nil*, for, in fact, there is just as much displacement, but being in the direction of the whole line, it is not evident.* The reader, prism in hand, should test all these statements as he proceeds.

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* It will be noticed that a long straight line, viewed through a prism, held about an inch from the eye, with its base-apex line at right angles to it, appears curved, with its concavity towards the apex. This is explained in Chapter XIV.

Apparent Displacement of Objects.—Since the apparent position of an object in space is determined by the direction in which the rays of light from the object enter the pupil, it follows that, when viewed through a prism, the object will appear misplaced by an angle which is nearly identical with the deviating angle of the prism.

In *Fig.* 28 the real position of the object is at *O*, but it appears to be at *I*, because the pencils of light enter the pupil as though they came from *I*.

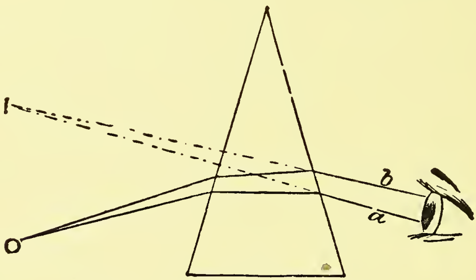


Fig. 28.

Displacement of the Visual Axis.—Since an eye likes to be diverted towards the (apparently) new position of the object, it follows that *the line of fixation is also displaced towards the edge of the prism by an angle equal to that by which the object appears displaced.* *Fig.* 28 illustrates how both an object and the eye that looks at it are displaced towards the edge. The displacement of the object is apparent, and that of the eye real. In *Fig.* 29

a prism is placed before one eye, *the other being covered*, and it is evident that the object (*O*) appears displaced to *F*, and the line of fixation (*v*) is diverted from *O* to *F*—both in the direction of the apex (*a*). The line (*F C*) in which the false image lies is called the "line of projection," since it marks the direction in which the picture on the retina is mentally projected.

Overcoming of Prisms.—With *binocular* vision, as in *Figs.* 30 to 33, the conditions are not quite

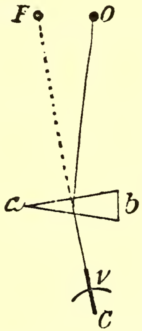
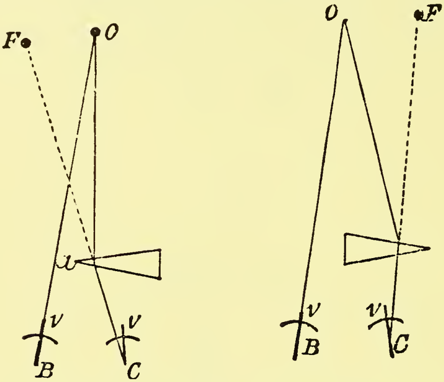


Fig. 29.—A prism before an eye, in monocular vision.

so simple. In all these figures the continuous lines show the actual course of the rays from the object (*O*), and the dotted lines are the lines of projection. They almost explain themselves: in all it will be seen that the object appears displaced towards the edge of the prism, though in some (*Figs.* 30 and 31) there is diplopia, since the object is seen in its true position (*O*) by the naked eye at the same time that its false

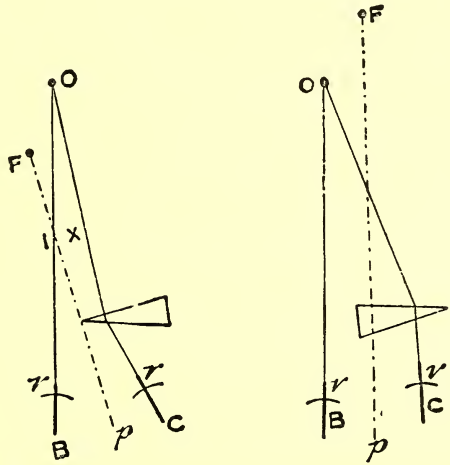
image (F) is seen by the other in the direction of the edge of the prism, set *in* in *Fig.* 30, and set *out* in *Fig.* 31. The first is called a converging prism, because it increases the convergence above the amount which is naturally associated with the accommodation for the object (O). The second is called a diverging prism, because it makes the convergence less than



Figs. 30 and 31.—Prisms not overcome, and therefore causing diplopia.

natural for the object looked at. In these two figures the conditions are those in which a prism is not "overcome." The desire to unite the true and the false image into one is not great enough to cause the visual axis (v) of the right eye (C) to accommodate itself to the new direction of the pencils of light from the object; or, to put it another way, the association between accommodation and convergence is not elastic

enough to allow the eyes to accommodate for *O*, and yet at the same time converge for a much nearer point (*Fig.* 30) or a much farther point (*Fig.* 31). There follows, as a necessary consequence, diplopia. The eyes have, as it were, to choose between diplopia and the maintenance of a forced abnormal relation between convergence and accommodation. With weak



Figs. 32 and 33.—Prisms overcome ; no diplopia.

prisms they prefer the latter, but with prisms above a certain strength they must perforce submit to the former.

In *Figs.* 32 and 33 the prisms are " overcome," and there is no diplopia ; it has been corrected by the desire for single vision, and the two images are fused into one. Now let us observe an interesting

fact. The *united* image appears displaced to a mean position midway between the true position of the object and that of its false image seen through the prism when the naked eye is shut. In *Figs.* 32 and 33 there is no diplopia. It is corrected by a rotation of the eye which lies behind the prism so as to receive again upon its fovea centralis the picture of the object which the prism had dislodged to another part of the retina; thus in these two figures the line of fixation (*v C*) has come to be in line with the rays of light coming through the prism from the object, and since both eyes receive a picture on the fovea, vision is single. In *Figs.* 30 and 31 the right eye is fixing nothing, and it is clear that the picture must fall on the retina away from the fovea; though when an effort is made to unite the images and bring the axis of vision (*v C*) into the line of the rays, the position of the picture on the retina will vary, and the images will move towards or from each other.

Prisms set with their edge inwards, as in *Figs.* 30 and 32, are called *converging prisms*, and those with their edge outwards, as in *Figs.* 31 and 33, are called *diverging prisms.* The former, when vision is binocular and without diplopia, by increasing the necessary convergence of the eyes, make objects appear nearer than they actually are, and the latter make them appear too far away. Thus, in *Fig.* 32, the real object (*O*), appears to be

at *F*.* As Hering has shown, convergence occurs as if for the cross, and both eyes are turned towards *I* by an impulse of the conjugate innervation which turns the two eyes to the left: it is the mental cognizance of *this* effort which imparts corresponding obliquity to the line of projection (*p F*). The prism, therefore (and this is important), does not test the strength of either rectus, since convergence is a single action affecting both eyes equally, neither does it measure the adduction or the abduction of the eyeballs, for these are measurements of what is called "the motor field," but it estimates by how much convergence can be made to exceed accommodation, or *vice versa*. The two functions are so associated in daily work that there are limits to the amount by which either can be increased or diminished without the other. Prisms should be placed as close as possible to the eye, then their angles of deviation nearly express the diplopia, or corrective squint, they occasion.

The strongest converging prisms that can be worn, short of producing diplopia, measure the prism-convergence, and the strongest diverging prisms, subject to the same condition, measure the prism-divergence.

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* Theoretically the combined image should be at *I*, were convergence the only criterion, but accommodation remains for a greater distance, and the "knowledge of distance" removes its apparent position from *I* to *F*.

One of three events, therefore, may happen on placing a prism before an eye. (1) If vision is monocular, one false image is seen, whose distance from the actual position of the object is unvarying, and dependent entirely on the deviating angle of the prism, *Fig.* 29. (2) With binocular vision the prism may be too strong to be overcome, in which case there is diplopia with an inconstant distance between the images, which approach each other spasmodically with every effort to overcome the prism. (3) The diplopia may be overcome and the object appear single, though its position in space is misjudged through an angle practically equivalent to half the deviating angle of the prism.

CHAPTER VII

COMPOSITION AND RESOLUTION OF PRISMS.

Composition.—We sometimes wish to combine one prism with another permanently, in the form of a spectacle lens, as, for instance, when a patient has hyperphoria as well as esophoria or exophoria.

Prisms can be compounded in the same way as forces if we draw straight lines proportional in length to the deviating powers of the prisms, and arrange them with due respect to the directions

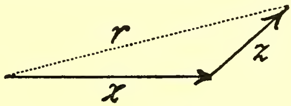

Fig. 34.

in which the apices of the prisms point. Thus, for example, let the line x in *Fig.* 34 represent the deviation of a horizontal prism, the arrow head indicating that the apex of the prism points to the right. Should we wish to combine with this a prism of half its strength, with its apex pointing 45° upwards and to the right, surmount the arrow head by the line z, drawn half as long as the first line and pointing 45° upwards and to the

right. Then the dotted line (r) connecting the free ends of these two represents, both in amount and in direction, the deviating power of the resultant prism which is capable of replacing the two.

In practice we rarely need to compound any prisms other than vertical and horizontal. Those who wish to avoid even the simple formulæ involved can do so by the graphic method

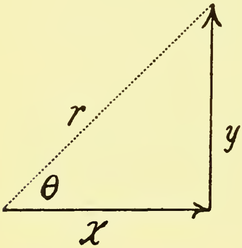

Fig. 35.

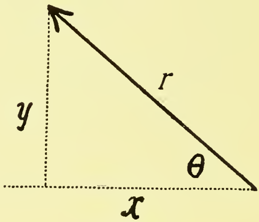

Fig. 36.

described in the first edition, and illustrated in *Fig.* 35. Draw a horizontal line as many inches long as there are degrees (or centunes) in the deviating angle of the horizontal prism; note which end of this line represents the apex of the prism and erect thereon a vertical line as many inches long as there are degrees (or centunes) in the deviating angle of the vertical prism to be compounded, should its apex point upward. If its apex be directed downward, the vertical line should be dropped downward instead. In either case the number of inches

between the free ends of these two lines equals, to a close approximation, the number of degrees (or centunes) in the required resultant prism, the base-apex line of which coincides with the direction of the dotted line, so that its inclination to the horizontal is easily measured by a protractor or graduated arc.

Should calculation be preferred, it is extremely simple.

Since in right-angled triangles, the squares on the two sides adjacent to the right angle are equal to the square on the hypotenuse—

$$r^2 = x^2 + y^2$$
$$\therefore r = \sqrt{x^2 + y^2}$$

The measure of the resultant prism therefore is the square root of the sum of the squares of the two prisms. As for the inclination of the base-apex line, it is evident that—

$$\tan. \theta = \frac{y}{x}$$

To put this into words, the angle of inclination from the horizontal is that whose tangent is the deviation of the vertical prism divided by that of the horizontal prism. It is evident that—

$$r = \frac{y}{\sin \theta}$$

Resolution.—Should a patient present himself wearing an oblique prism, we naturally wish to know its vertical and horizontal components. It is fairly easy to do this by the usual optical

tests, employing a horizontal and a vertical line on the wall to carry them out. But should the prescription state the strength of the oblique prism we might prefer to resolve it into its horizontal and vertical components graphically or by calculation. All that is requisite is to draw a straight line to represent the base-apex line of the oblique prism, and having the same inclination from the horizontal. From that end of the line which represents the apex, drop a perpendicular to meet a horizontal line drawn from its other end as *y* on *x* in *Fig.* 36.

Then *y x* represent the deviations of the vertical and horizontal components respectively; the units of length in each represent the degrees of deviation as compared with the number of similar units in the line which represents the oblique prism.

Here again it is evident that—

$$\sin.\ \theta = \frac{y}{r}$$

$$\therefore\ y = r \sin.\ \theta$$

$$\text{and } x = r \cos.\ \theta$$

CHAPTER VIII

ROTATING PRISMS

We are indebted to Sir John Herschel for showing how, by placing two prisms in apposition, and rotating them in opposite directions, we can produce the effect of a single increasing prism. If two prisms of equal strength be placed with the apex of each against the base of the other, they so neutralize each other as to have the effect of only a thick plate of glass. The more they are rotated from this position, the greater is the prismatic effect of the combination, till it reaches its height, when the two apices coincide.

Crétès, of Paris, mounted two such prisms in a circular frame with a handle, so that on moving a button along the handle the prisms revolved in opposite directions at equal rates. This useful instrument is commonly known as "Crétès' prism," or the "prisme mobile." Each prism has a refracting angle of 12°, and the instrument is marked accordingly. Landolt has had it marked in degrees of deviation, and metre angles. If wishing to know how strong a prism can be borne before one eye without creating diplopia, we start at zero, and press up the button till

double vision commences, when the strength of the prism can be read off.

The neatest form in which Herschel's prisms are put up is that of the "Rotary prism," designed by Dr. S. R. Risley, of Philadelphia. It is small and circular, and can be placed in an ordinary trial-frame. On turning the little button the two prisms, each of 15°, rotate in opposite directions. The diameter in which the button lies coincides with the base-apex line of

Fig. 37.—Risley's Rotary Prism.

the *virtual* prism, however much the actual prisms are rotated. The virtual prism can be increased from 0° to 30° (15°*d*). The markings indicate the physical angle of the virtual prism, not the deviating angle. If the apex of the front actual prism be to the button side of zero, the position of the button indicates the apex of the virtual prism; if the actual apex be to the other side of zero, the button indicates the base of the virtual prism.

Every form of Herschel's prism, however, has one disadvantage. The diminution in the visual acuity, occasioned by the chromatic aberration and prismatic astigmatism, is in one eye only, and thus the impulse to fusion is lessened more than if it were equally distributed over both eyes. To rectify this fault the author proposed and figured in the first edition a design by which the twin prisms, instead of rotating against each other's faces, should be placed one before each eye in a spectacle frame, so that they can still rotate in opposite directions, and produce the same binocular effect. The design was difficult to manufacture, and only quite recently has it been actually made by Mr. Dixey, of New Bond Street. Each prism is attached to a toothed wheel geared on to the thread of an endless screw attached to the horizontal bar, which can be rotated by the milled head, as in *Fig.* 38. The instrument allows us to modify the positions of the visual axes in their relation to each other, both horizontally and vertically.

For the former, that is to effect greater convergence or divergence of the visual axes, we start with each prism edge up. Under these conditions both visual axes are raised a little, and the patient's head is unconsciously thrown slightly backwards to compensate for this, but convergence is unaffected. On rotating the milled head, the edges of the prisms become inclined either towards

or away from each other. In the first case they cause convergence, and in the second divergence of the visual axes to any required amount within the power of the prisms.

To effect *vertical* separation of the visual lines, a slight readjustment of the apparatus forms a needful preliminary. One prism must be

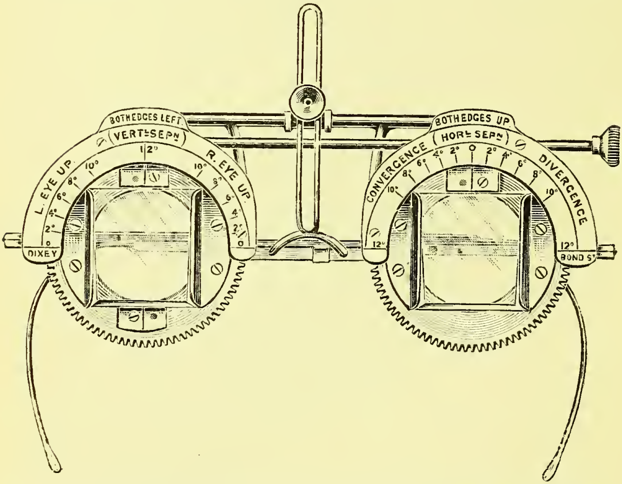

Fig. 38.—Author's prism-verger, with prisms before both eyes.

taken out and set with its edge downwards instead of upwards, after which both are rotated in opposite directions until each edge comes to point to the left. When thus set they have no effect on the mutual relations of the visual lines whether horizontal or vertical. It is true that both eyes are turned slightly to the left, to allow for which the patient's head is turned

proportionately to the right. On rotating the prisms, however, away from this position, one eye is made to rise and the other to fall—which eye rises and which falls depends on the direction in which the milled head is turned. To save calculation and prevent mistakes, I have marked every necessary detail on the graduated arcs above the prisms. Full directions for its use will be found in Chapter X.

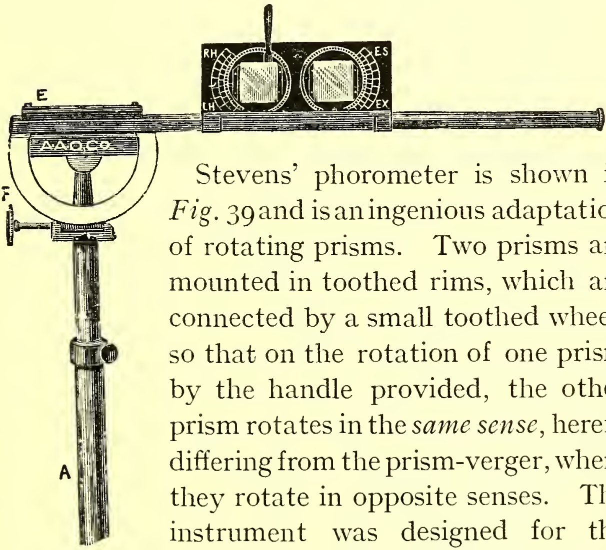

Fig. 39.

Stevens' phorometer is shown in *Fig.* 39 and is an ingenious adaptation of rotating prisms. Two prisms are mounted in toothed rims, which are connected by a small toothed wheel, so that on the rotation of one prism by the handle provided, the other prism rotates in the *same sense*, herein differing from the prism-verger, where they rotate in opposite senses. The instrument was designed for the measurement of heterophoria, and is the best mechanism for the purpose, though methods requiring no mechanism are freer from any possible instrumental defects.

CHAPTER IX

THE USE OF PRISMS IN THE DIAGNOSIS OF HETEROPHORIA

WE are indebted to Stevens for replacing older titles for suppressed or latent squints by a convenient nomenclature. Heterophoria is his generic name for the whole class: hyperphoria for latent upward deviation of an eye; exophoria for latent divergence, and esophoria for latent convergence. The term cyclophoria, introduced by Savage and Price, may be conveniently added to indicate latent torsion, or wheel-motion of the eye about its own fore-and-aft axis.

Heterophoria has, however, been looked upon too much in the past as a homogeneous entity, without sufficient enquiry being made as to its intrinsic varieties, of which at present we may recognize five. These are:—

1. *Structural* heterophoria, due to anatomical peculiarities in the structure or insertion of the muscles. This is one of the least common varieties.

2. *Tonic* heterophoria. This is the common kind revealed in distant vision by rod tests,

phorometers and such like, in emmetropes or corrected ametropes. The tonic equilibrium does not, as we have seen elsewhere, represent the anatomical position of the eyes, but their physiological position.

3. *Accommodative* heterophoria. Under this heading falls that which is due in distant vision to the existence of hypermetropia, and in near vision to either that or to paresis of the ciliary muscle.

4. *Paralytic* heterophoria. This is not looked for sufficiently, and is not infrequently missed. To discover its existence, the usual tests which are made should not only be instituted with the eyes looking straight forward, but also with the patient's face turned in different directions away from the object looked at, so as to test the balance in the various areas of the motor field with which we are so well acquainted in measuring manifest paralyses. Should the amount of heterophoria increase on looking in any one direction, it is quite easy to detect which is the weak muscle by following the same rules as those given for detecting manifest paralyses. (*Vide* "Ocular Muscles," Chap. VIII.)

5. *Spasmodic* heterophoria. This is mostly due to reflex causes, or to hysteria, and if it be due to spasm of a single muscle, introduces an inequality of the amount of deviation in different parts of the motor field. I am convinced,

however, that this is extremely rare, and the only common spasmodic condition is that of the conjugate innervation of convergence.

Reflex inhibition of the same innervation is, I think, possible, though difficult to distinguish from hyperkinesis of the diverging centre, if such centre exist.

Von Graefe was the first to suggest the use of prisms for the purpose of so dissociating the eyes as to allow their natural balance to become manifest. By placing a vertical prism before one eye, strong enough to create vertical diplopia, the reflex stimulus to fusion is suppressed, and the eyes assume what Von Graefe called their "position of equilibrium." A prism of four degrees ($2^\circ\ d$) is generally strong enough to create vertical diplopia which cannot be overcome in distant vision, but for near vision a stronger prism must be used (Chap. XVIII). Von Graefe used one of 15°. The distant object which he employed was a candle flame, the false image of which lay directly above or below the flame when the equilibrium was perfect, while lateral displacement of a kind to produce homonymous or heteronymous diplopia indicated respectively relative convergence or divergence. For near vision his selected object was a dot on a card with a vertical line through it. The prism reduplicated the dot, but only lengthened the line. The disadvantage of this pretty device was that the

overlapping portions of the line were sufficient to maintain the exercise of fusion, and thus mask any tendency to latent deviation that might exist. It was only considerable insufficiencies that manifested themselves by this test. The *degree* of deviation Von Graefe measured by finding what additional horizontal prism was required to bring the candle flames of the distant test into one vertical line, and to unite the dots and lines of the near test device. Without using a second prism, however, the first prism may be rotated till one image is over the other, and then from the strength of the prism, and the amount of rotation, a calculation can be made of the deviation of the eyes.*

Since Von Graefe's near and distant tests both require the head to be erect and the prism to be held strictly vertical, they give false results unless these conditions are complied with. The slightest inclination of the base-apex line of the prism introduces a lateral displacement of the false image, which we may easily mistake for a latent

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* The formula is as follows, where d is the deviating angle of the prism, ω its angle of rotation, and H the angle by which the eye deviates, and by which, therefore, the false image is misplaced,

$$\text{Tan. } H = \text{Sin. } \omega \text{ Tan. } d.$$

In words, the tangent of the horizontal angle we wish to measure, through which the false image is misplaced, is the multiple of the sine of the angle through which the prism has been rotated, and the tangent of the angle by which it deflects rays of light. Or, more simply,

$$H = d \text{ Sin. } \omega.$$

deviation where none may exist. To remedy this I made use, several years ago, of a double prism, which is shown in *Fig.* 40. It is composed of two prisms, each of 2° or 3° *d* united by their bases. The patient, shutting the left eye, holds this prism before the right one and looks through it at a flame, two images of which are seen, one above and one below the real position of the flame. Nothing is easier than to place the

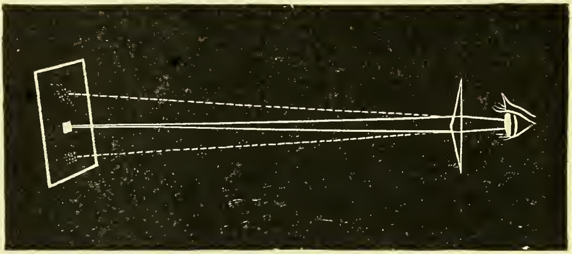


Fig. 40.—A double prism.

prism so that the two images appear vertical. On opening the left eye the flame is seen mid-way between its two images, and in the same vertical line if there be no latent deviation. The double prism made the test more delicate than a single one, but later it was replaced by the still better plan of a glass rod.*

* The rod test was suggested to the author by a faint streak of light seen uniting the two false images, due to the fact, shown by Mr. Berry, that the edge of the obtuse prism is not a mathematical line, but from imperfect manufacture, a rounded ridge. It was easy to deduce from this that a glass rod would produce a better streak of light, by acting as a strong cylindrical lens (*Ophth. Review*, May, 1890).

The first form of this little instrument was that shown in *Fig.* 41, consisting of a short glass rod, mounted in a metal disc. It constituted a new departure as being the first of the little series of devices since introduced by others in which dissociation of the eyes is effected by altering the *shape* of one of the images instead of, as formerly, displacing it; such as Stevens' stenopæic lens, which causes a circular diffusion-patch

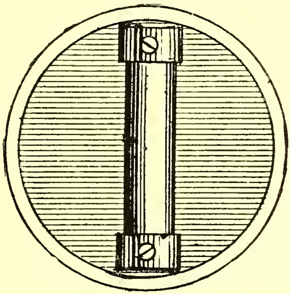


Fig. 41.—The first form of the glass-rod test.

of light instead of a diffusion line, and Rogers' cone (*Fig.* 43), which produces a circular band of light. The latter is manufactured by Raphael. A convenient modification consists of a series of glass rods, placed parallel and in apposition, and united at their ends by sealing wax. This form anyone can make for himself by breaking up a thin glass rod into several pieces an inch long, laying them together on a smooth level surface, and sealing their ends together with sealing wax; but a better form of mounting is shown in *Fig* 42.

A distant flame, with this before one eye, appears converted into a long streak of light, which there is no temptation to regard as a false image of the flame, especially if red glass be used. If the streak pass through the flame, equilibrium is perfect, but if otherwise, its distance indicates the amount of latent deviation. The prism that is able to bring the line and the flame together is the measure of it. A very convenient plan, in the absence of a tangent scale (*Fig.* 44), is to place the rod test in

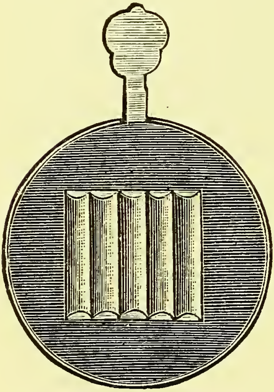


Fig. 42.

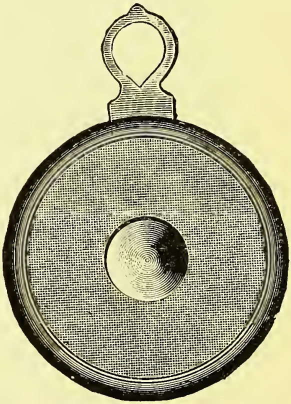


Fig. 43.

one side of a trial frame, and Risley's "rotary prism" in the other, and turn the latter till the line crosses the flame. Better than either, however, are the scales graduated in tangents of degrees, arranged by the author for the purpose and shown in *Fig.* 44, since the patient can read at any moment the figure crossed by the line of

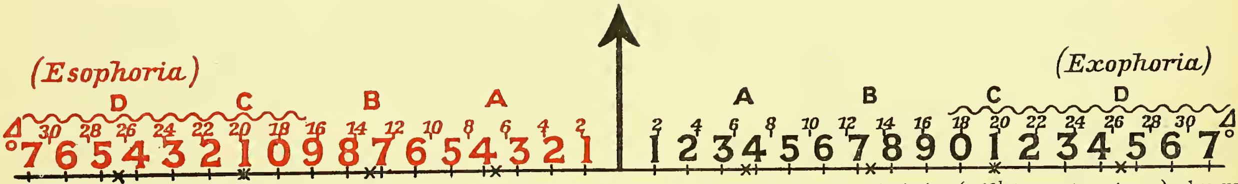


Fig. 45.—Hold this arrow-scale a quarter of a metre from the eyes with a prism of six degrees deviation (=$10\frac{1}{2}$ tangent-centunes) edge up before the right eye. The large figures indicate degrees, the small figures tangent-centunes (=prism-dioptres) while A, B, C, D are metre-angles.

light, and thus variations can be followed. The full extent of a latent deviation does not occur at once, but the line gradually moves farther from the flame. Such a scale, graduated for use at 5 metres, and with directions for use, can be obtained from Messrs. Curry & Paxton: it is also furnished with smaller figures for the objective measurement of squint, but that does not belong to the subject before us.

For near vision, an arrow-scale was constructed by the author many years ago,* and published

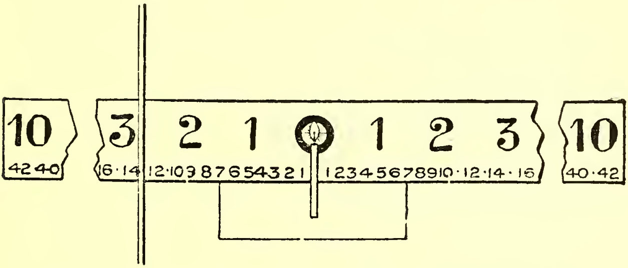


Fig. 44.—Tangent scale for use with the glass-rod test.

by Messrs. Curry & Paxton. It is graduated in degrees and metre-angles, for ¼ metre, with a vertical arrow running up from zero, as in *Fig.* 45, such that when reduplicated by a square vertical prism held before one eye, the lower arrow points to that figure on the upper scale which measures the deviation. It is best to use a prism of 12° (6° *d*), permanently mounted in

* *Syme Surgical Essay*, 1884.

an ordinary strong cataract frame, which can be placed on the patient in a moment, and needs no hand to hold it. A thread, $\frac{1}{4}$ metre long, should be permanently fixed to the card to adjust its distance from the patient correctly. It is provided with a printed sentence, as well as the figures, to ensure full accommodation—an improvement suggested by Mr. Berry.

Convenient holders for the above-mentioned arrow-scale device have been made by Messrs.

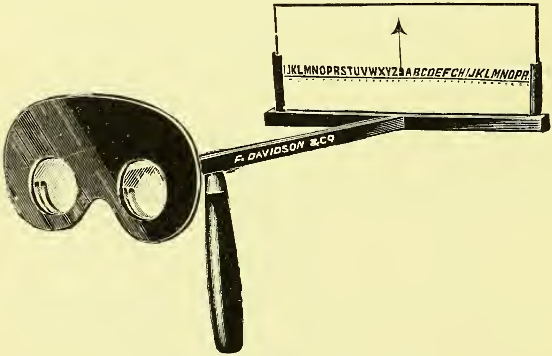


Fig. 46.—Holder for arrow-scale.

Raphael, and by Mr. Davidson, of Great Portland Street. The latter is shown in *Fig.* 46.

Tangent scales can be made for any intermediate distance, but though I have used them for physiological investigation, they are not necessary in practice. If graduated in degrees or metre angles, a different scale is of course needed for each distance; while if marked in

centimetres, and provided with an arrow capable of being lengthened or shortened, one is sufficient. At one metre each centimetre would stand for a centune ; at half a metre for two centunes ; and so on.

What is the due significance to attach to the near vision test ? It clearly does not decide the strength of one rectus, for convergence is a single function affecting both eyes equally. It is true that each rectus is a link in the chain of convergence, and *may* be the weak link, but that would show itself best in testing the lateral movements of the eyes. Neither does it afford a pure test of the strength of convergence, for the function of convergence may be quite normal, and yet considerable exophoria appear in the test for latent equilibrium, from one of its stimuli being wanting, as in myopia, where accommodation is slight, or absent.

HYPERPHORIA is of two kinds, paretic and concomitant. The name "simple hyperphoria" should be confined to the latter. If the deviation be of the same amount on looking upwards or downwards, it is concomitant ; if otherwise, paretic.

Von Graefe's original way of disclosing these vertical deviations was by placing before one eye a prism, with its edge outwards, and slightly stronger than can be overcome—say one of 6° *d*. In this test especially, precision in the setting of

the prism is essential, since imperfect setting, or an inclination of the patient's head, causes apparent hyperphoria, when such does not exist, as shown in *Fig.* 21. Slight vertical deviations are less frequent and more important than slight horizontal ones, since in all normal individuals the vertical equilibrium is perfect. The glass rod is a delicate test for it, free from the error introduced by imperfect setting of prisms, since slight accidental displacements of the rod, or of the head, do not appreciably affect the distance of the line of light from the flame. The source of illumination should be either a small electric lamp, preferably with an iris diaphragm, or a gas-jet turned down to about a quarter of an inch in height, at the distance of 5 or 6 metres, while a piece of green glass held before the other eye greatly improves the test by subduing the illumination of the flame, which otherwise is apt to eclipse the line by its superior brightness. The author finds it very convenient to employ a cataract frame furnished with red multiple rods before one eye, and green glass before the other. Care should be taken that direct light from no other source, such as a window or a fire, fall on the glass rod, and this can be ensured by standing so as to place it in one's own shadow : a dark room is not necessary. The measure of the hyperphoria can be taken by finding what prism brings the flame and line together, or how much

rotation must be given to Risley's "rotary prism," but a scale of degrees, as shown in *Fig.* 47, is what I greatly prefer. Like the other scales, it is obtainable from Messrs. Curry & Paxton.

For *near* vision it is not so easy to measure hyperphoria, nor is it so often required. (*a*) A

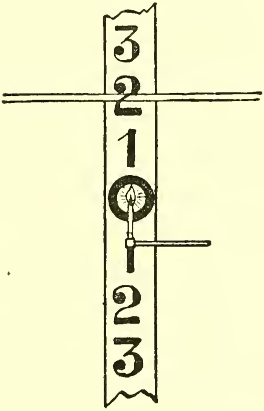


Fig. 47.—A tangent scale to measure hyperphoria with a glass rod. It may be combined with that of Fig. 44 in the form of a cross.

prism can be held before one eye with its edge outwards, while a dot on a card is looked at. The card should have degrees marked upwards and downwards from the dot. (*b*) A more accurate test, however, is afforded by the use of the glass-rod test after replacing the dot by a tiny electric lamp shining through a hole in the card. (*c*) I used to use a diminutive silver sphere like a globule of mercury, on which the reflection from

the window was bright enough to yield a streak if viewed through a special form of rod test which economizes the light. This consisted of a plano-convex cylindrical lens with a radius of curvature of about 10 mm. It is to be held with its plane surface next the eye, and ushers the maximum amount of light into the pupil. Incidentally the interesting fact may be mentioned that this little device enables the stars to be used for testing the balance of the eyes with very distant vision, since a very bright and well-defined streak is obtainable from any brilliant star. (*d*) A stereoscope marked before one eye with a vertical line graduated in degrees up and down from a central zero point, and before the other eye with a horizontal line exactly level with the zero point just mentioned. On looking into the instrument, the horizontal line appears to be across the zero of the vertical one if equilibrium is perfect, but if not, the figure crossed by the horizontal line measures the hyperphoria. The stereoscope, however, is not free from error if not held perfectly straight. (*e*) The author's latest plan is to use a piece of cardboard with a small perforation in its centre, behind which a right-angled prism is cemented to reflect light from the sky through the aperture. This enables the glass rod or cylinder to be used.

Instances of hyperphoria are not so rare as they are thought to be. It is well, when opportunity

offers, to test them again after a considerable interval to see if the amount is stationary: often their persistency is remarkable, and the same case will show, year after year, exactly the same measurement. Even slight hyperphoriæ sometimes cause great inconvenience, and even headache and asthenopia. On the other hand, hyperphoria may be very considerable without causing a single symptom, or seeming to be of any consequence. I might instance the case of a young man, 18 years of age, who had right hyperphoria of nearly 10°, without any headache or discomfort from it. His right eye was noticed by his mother occasionally to "roll up," and since his grandmother was said to exhibit the same phenomenon, it is more than likely that there was hereditary transmission of the defect, and its toleration would thus be accounted for. This case called for no treatment, since the anomaly caused no discomfort; tenotomy of the right superior rectus, or advancement of the left inferior, would have been indicated had distressing symptoms been present.

Monocular and Double Hyperphoria.—In true hyperphoria one eye turns up when it is covered, as much as the other turns down when it is covered, but Mr. Berry first noticed some rare aberrant forms of hyperphoria, in which one eye turns up without the other turning down, or in which either eye turns up when covered.

Paretic Hyperphoria.—In this the latent deviation is greater in some directions than in others, and by using the rod test, with the patient's head in different positions, it is easy to discover which eye is at fault, and which muscle is affected.*

* *Ophth. Review*, vol. ix. p. 287.

CHAPTER X

PRISM-RANGE OF FUSION

THE cerebral act of uniting the two brain pictures of simultaneous retinal images in the two eyes has been called, by German physiologists, "fusion." Older writers were accustomed to speak of the "desire for single vision" or "nature's abhorrence of double images."

The measure of the desire for fusion varies in different individuals, as also the power of effecting it, and a distinction has to be made between these two factors. The desire varies very much with the amount of attention directed to the double images. When the mind is occupied elsewhere, diplopia may be tolerated, which at once disappears when the attention is arrested by the images.

By the "breadth" or "amplitude" of fusion is meant the greatest artificial diplopia capable of being overcome; or, in other words, the greatest attainable departure of the visual lines from the object looked at while still preserving single vision. When measured by prisms it will be convenient to call it the prism-range of fusion.

The power of overcoming prisms has

been discussed in a previous chapter. The strongest prism capable of being overcome with its edge pointing in some definite direction, measures the breadth of fusion in the meridian indicated by its base-apex line. It is evident that a distinction should be made between tests for distant vision, and which therefore involve parallelism of the visual lines to begin with, and tests for near vision where the visual axes converge to start with. The breadth of fusion is in practice found to vary under these two conditions.

Prism-divergence.—With distant vision the eyes can normally overcome a prism, edge outwards, of 4° deviation, i.e., of 8° edge-angle. It must

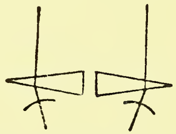


Fig. 48.—Prism-divergence in distant vision.

not be imagined that a prism of this kind measures the strength of the external rectus, for it only invokes a slight contraction of that muscle, and but a fraction indeed of its full power of turning the eye toward the outer canthus. To measure the strength of the muscle *per se*, we should have to test its power of complete abduction toward the temple. The prism affords a measure of the diverging faculty resident in the higher centres.

The artificial divergence produced by the prism has been aptly called by Duane the *prism-divergence*. The amount of this prism-divergence measures the competency of the conjugate diverging innervation, together with that which incites it, namely, the strength of the fusion desire. In one person the desire may be keen, because the eyes are of equal visual acuteness, and yet through neurasthenia, local or general, the power of fulfilling the desire may be subnormal. In another person the converse may be true; the constitution may be robust, and the local vigour of the co-ordinating centres may be normal, and yet the desire for fusion may be deficient, either through a congenital feebleness of the centres provided for it, or through defective vision of one eye.

There is one other important factor which has also to be considered, and that is the starting point or the position of rest of the eyes, or, in other words, the amount of their departure from orthophoria. In eyes which have considerable esophoria greater effort is required to overcome a diverging prism than where exophoria allows a certain amount of divergence before any effort is put forth at all.

In ametropia, if lenses of sufficient strength are placed in a trial-frame, the experiment can be made by separating them if concave, or approximating them if convex, till double vision

occurs, and then the prismatic effect can be calculated at the rate of one centune per dioptre-centimetre (just as in mechanics we speak of foot-pounds), or more accurately, if required, from the author's rule i, p. 182, or the tables by Dr. Percival.

Thus, if the distance between the optical centres of lenses of –4 D be increased till it is 16 mm. greater than the distance between the centres of motion of the eyes, we divide the 16 mm. between the two eyes, for clearly each lens is displaced 8 mm., and therefore there is, for each eye, 1° 40′ of divergence.

The average prism-divergence is greater in myopia and less in hypermetropia, according to Schuurman.

Prism-convergence.—The power of overcoming converging prisms, i.e., prisms with their edges inwards, while distant vision is in exercise, is far greater than that of overcoming diverging prisms. The measurement of prism-convergence requires special precautions. It is more easy to converge when we accommodate as well, since the impulse to the one act stimulates the other. To guard against this, the object looked at should be provided with test types small enough to represent the visual acuteness of the patient at that distance, and the measurement should be recorded as soon as the acuteness becomes in the least impaired. Moreover, the prism-convergence

is so rapidly increased by practice that the information afforded is of comparatively little value. At the first trial two prisms, each of 6° deviation, should be easily overcome, and sometimes prisms of two or three times that strength can be conquered.

The power of overcoming vertical prisms is of more interest. It measures the power of vertical separation of the visual lines under the stimulus of threatened diplopia. To measure the right supravergence the prism should be held edge up before the right eye, or edge down before the left; and for left supravergence, edge up before the left eye or down before the right.

Here again it is important to observe what is so often overlooked, namely, that it is not merely the strength of the elevators or depressors that is estimated, for that is a problem of the motor field, measured by supraduction and infraduction, or, in other words, the greatest possible elevation or depression of the cornea. Prism-supravergence falls precisely into line with prism-convergence and -divergence, and is strictly concerned with co-ordinating innervations which exist in order to keep the visual lines true to the visual plane which passes through the centres of motion of the two eyes and the object looked at.

The normal amount of supravergence is represented by a prism of 1° *d*.

So far we have spoken of a single prism held

before one eye. Since, however, chromatic aberration and prismatic astigmatism slightly impair the image seen through a strong prism, the fusion desire is lessened thereby, and truer results are obtained by dividing the prism into two of half the strength, one placed before each eye. Diverging prisms of this kind would have

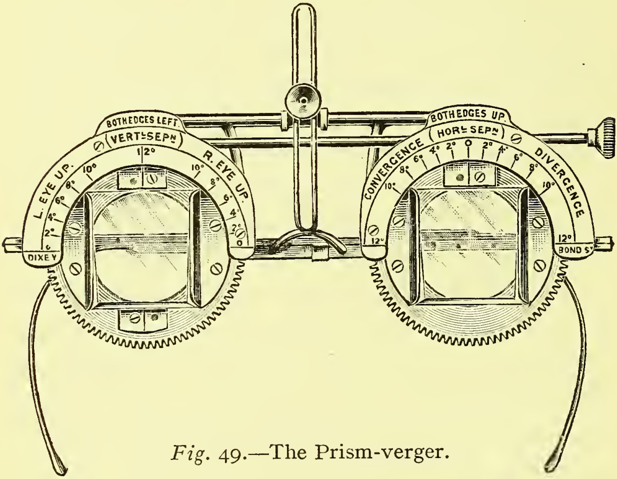
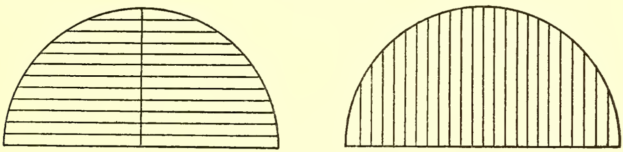


Fig. 49.—The Prism-verger.

both their edges outwards; converging prisms, both edges inwards; supraverging prisms, one edge upwards and the other downwards.

The difficulty found in practice of working with two prisms thus placed suggested to the author the apparatus already referred to in Chap. VIII. and shown in *Fig.* 49. Two prisms are so mounted in a frame as to rotate in opposite directions at strictly equal rates.

Each prism has a deviation of 6° *d*, but while the right prism is permanently attached to the cog-wheel in which it is set, the left prism is reversible; that is, it can be drawn out of its groove with its base, for example, downward, and can be replaced with its base upward. The object of this arrangement is to allow the vertical balance of the eyes to be measured as well as the horizontal. It is to be noted that of the two graduated arcs each has a special purpose with reference, not specially to the prism over which it is placed, but to both prisms jointly. It is



No. 1. *No.* 2.

Fig. 50.—No. 1 shows how to graduate an arch for the prism-verger with "both edges left." No. 2 how to graduate with "both edges up." The radius in each case being divided into 12 equal parts.

only for convenience that each arc is placed over one prism. One arc relates purely to the horizontal deviations of the eyes, and the other relates purely to their vertical balance.

(*a*). To measure the *prism-divergence*, rotate the milled head till the edges of both prisms point upward. Under these conditions the patient's eyes are raised a little, or probably his head thrown

slightly back. The relations of the eyes to each other are unaffected. On rotating the milled head, however, the edges can be made to part from one another so as to cause prism-divergence of the eyes. The patient should be regarding a distant light or candle with lively attention, prepared to exclaim at the first appearance of diplopia. The movement should be made steadily, and the result is much more definite than when a series of prisms are tried with considerable gaps between them. The amount of prism overcome is read off the quadrant marked "Divergence." About 4° is the average, i.e., 2° *d* for each eye as first stated by Stevens.

(*b*). To measure the *prism-convergence*, rotate the milled head in the opposite direction to the last test, but direct the attention this time to distant types sufficiently small to measure the patient's visual acuteness. It may very likely be found that the rotating prisms are not strong enough to measure the whole range of fusion in this direction. A second ordinary trial frame containing strong converging prisms can be placed behind it if desired, or better still a narrow cell for accessory stationary prisms may be provided behind the rotating ones. The amount of prism overcome is read off that quadrant marked "Convergence."

(*c*). To measure the *vertical* breadth of fusion (prism-supravergence), start with the edge of

each prism looking to the left. In this position the relations of the two eyes to each other are not affected, though both will be turned slightly to the left, and the patient's head slightly in the same direction also.

It may be asked why the edges of the prisms are made to point to the left rather than to the right. I find that the eyes can be turned slightly to the left, and maintained thus with less fatigue than when diverted to the right. This is no doubt due to habit, since in handwriting the eyes are accustomed to left deviation for long periods at a time.

On now rotating the milled head, the edge of one prism will rise and that of the other fall; the rising edge lies in front of the cornea which is being raised. To raise the right cornea, therefore, we should rotate the milled head in the direction which makes the edge of the right prism ascend.

A distant flame, or a white letter or spot on black velvet, affords the best object for the patient to look at, and the amount of right supravergence can now be read off the graduated arc which is headed by the words "both edges left." The milled head should next be turned in the opposite direction, so as to raise the left cornea till the onset of diplopia bids us arrest the motion. The left supravergence is now read from the same graduated arc. The normal amount of supravergence of either side is 1° of

deviation, but it is often as much as 2° or more. The important feature is not the amount, so much as the equality of the right and left supravergence. In hyperphoria we naturally find greater prism-supravergence of the hyperphoric eye than of the other, the difference indeed being in theory exactly double that of the hyperphoria.

Fusion Range in Near Vision.—In the preceding part of this chapter distant objects have been employed. It is useful additionally to gather information about the prism range when near objects are regarded, and more particularly about the prism-divergence and the prism-convergence. To do this we may employ Von Graefe's design of a round dot with a vertical line through it, or, better still, a row of very fine letters arranged in a vertical line to ensure a fixed amount of accommodation while discovering to what extent convergence can be increased or relaxed while the accommodation remains unchanged. The value of the information gained is somewhat discounted by the greater prism strength overcome with a little practice, showing that we are not dealing with fixed quantities.

This test is easily made with the double rotating prism-verger, the same directions being followed as for distant vision.

By its use we find the strongest pair of diverging prisms, as shown in dotted outline in *Fig.* 51, and then the strongest pair of converging prisms,

as shown in continuous outline, compatible with single distinct vision of an object or type at some chosen distance, preferably that at which the patient's daily work is done (occupation distance), and for which spectacles are most required.

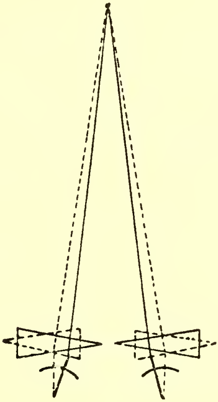


Fig. 51.—The prism range of fusion for near objects.

The diverging prisms measure the negative part of the range and the converging prisms its positive part. It will be noted that the former make the print look large, while the latter make it look small.

The following table shows the fusion range found for varying distances in a man of 32, with normal refraction, by the old method of a succession of single prisms. The first column (A) gives the refracting angle of the highest + and − prism he can overcome at the various distances. The second column (D) shows the deviation of *each* eye

TABLE IV (first edition).

	A. Single Prism	*D.* Deflection of *each* axis	*R.* Fusion Range
6 m.	+ 16° : − 3°	+ $4\frac{1}{2}$° : − $\frac{3}{4}$°	$5\frac{1}{4}$°
1 m.	+ 18° : − 8°	+ 5° : − 2°	7°
$\frac{1}{2}$ m.	+ 20° : − 12°	+ $5\frac{1}{2}$° : − $3\frac{1}{4}$°	$8\frac{3}{4}$°
$\frac{1}{3}$ m.	+ 24° : − 16°	+ $6\frac{3}{4}$° : − $4\frac{1}{2}$°	$11\frac{1}{4}$°
$\frac{1}{4}$ m.	+ 30° : − 18°	+ 9° : − 5°	14°

produced by the aforesaid prisms ; it is found by calculating the deviating angle of each prism, and dividing it between the two eyes. Thus the deviating angle of a prism of 16° is 9° *d*, and half this is $4\frac{1}{2}$°, so that at 6 m. each eye can converge $4\frac{1}{2}$° without losing the distinctness of an object. The third column (*R*) gives the fusion range for each eye, calculated from the figures in column *D*.

It will be noticed that both the positive and negative parts of the range increase as vision becomes nearer ; also that the positive part exceeds the negative, though less so as vision becomes nearer. The case cannot be taken as expressing the normal amounts, but the features just mentioned will, I think, be found to prevail. The table agrees more closely with Nagel's chart than with Donders'. The results obtained vary not only according to the peculiarities of the individual, but according to the method pursued. Thus, if we test the negative side of the range first, the positive side will be smaller than if we began with it first. As a rule, the limit of the

negative part shows itself by the appearance of diplopia, and of the positive part by indistinctness from commencing excess of accommodation. The impairment of distinctness by strong prisms is apt to be confounded with this latter, so it is not easy to get exact results. The use of the prism-verger, which has only been manufactured during the revision of this edition, will doubtless make it easy to arrive at an average standard.

Absolute Maximum of Convergence.—There is little to be gained in practice by the determination of the convergence maximum by prisms, though it is of some physiological interest.

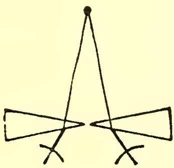


Fig. 52.—The absolute maximum of convergence.

When a finger is brought near between the patient's eyes until one eye begins to waver or deviate, we generally learn all we need to know, but the test is greatly wanting in precision. It does not give the "*absolute* maximum of convergence," for as soon as the point of fixation passes within the binocular near point of accommodation, diffusion circles appear on the retinæ, and increase rapidly in size with every approach, till the eyes resign the impossible task of

accommodating further—then the ciliary muscle relaxes, and with it the convergence.

This test depends largely, I believe, on the point of *the resignation of accommodative effort.* But were the accommodative efforts to persevere, it is scarcely to be expected that two pictures on their respective retinæ, each composed of a mass of diffusion circles, would excite the desire for fusion in its full strength, as clearly defined pictures are accustomed to do.

To measure more accurately, we should first discover the near point of accommodation, and while looking at an object at that distance from the double rotating prism frame, rotate the edges of the prisms inwards till diplopia commences.

By adding the maximum convergence thus found to the already ascertained prism-divergence for distance, we get the "absolute range (amplitude) of convergence" (a^c), of which, according to Dr. Landolt, not more than one third or one fourth can be continuously in exercise for comfortable vision.

CHAPTER XI

EXERCISING PRISMS

The oculomotor innervations can, like all others, be strengthened by exercise, and prisms are valuable for the purpose. It is not, however, the muscles themselves which are principally strengthened. The contractions required in their bellies for the perpetual excursions of the eyeballs all day long, are far greater than any we impose upon them by prisms. It is rather the co-ordinating centres which are strengthened, so as to readjust their relationships, and acquire increased functional power. It is interesting to observe that by wearing prisms we can alter the position of equilibrium of the eyes for some little time afterwards. Thus, if I wear converging prisms, amounting together to 11°*d*, for ten minutes, the rod test reveals, on their removal, $5\frac{1}{2}°$ of esophoria in distant vision, which takes a good many minutes to disappear. In near vision, under the same conditions, my usual exophoria of $3\frac{1}{2}°$ has been converted into an esophoria of the same amount. The experiment causes slight headache, not so much during the wearing of the prisms as afterwards, and especially when engaged in near

vision, on account of the fact that the correction of esophoria by the feeble diverging innervation involves greater fatigue than the correction of exophoria by the more powerful converging innervation.

Again, if I wear convex lenses for near vision for a few hours, the physiological exophoria at reading distance is, after their removal, less than before.* This is because of the training which the converging innervation has undergone in having to do its work with less support from accommodation. It explains why spectacles, which cause discomfort at first, become tolerated after a while. There are, however, limits to the effect of training, otherwise prisms would never be needed, except for pareses of ocular muscles. If there were no limits to training, there would never be a concomitant squint, for the training of the whole life, before the squint appears, is in the direction of overcoming it. Accommodation and convergence require definite groups of brain cells for their centres, but the mutual adaptation and correlation of these seem largely left to be perfected by *education*. "Circumstances cannot create a faculty, however much they may develop or retard its exercise, but we can conceive that faculties were created with a view to circumstances, and even capable, within limits, of being modified by

**Essay for Syme Surgical Fellowship*, 1884.

them."* The human body is thus made capable, within limits, of adapting itself to its circumstances, and in no part of it is this more beautifully seen than in the relative activities of the ocular muscles.

In neurasthenics the limits of adaptability may be more quickly reached, or adaptation may be attained only at the cost of much discomfort, while even in healthy individuals unusual conditions may make it advantageous to strengthen the process by prismatic exercises. The patients who require this help are generally those whose natural power of adaptation has already signally failed.

Dr. Dyer introduced what he called the so-called "invigorant plan" of treating heterophoria. It does not, if my belief be correct, so much invigorate the muscles, as train the powers of accommodation and convergence to assume broader relations to each other in their work. His plan consisted in giving the patient four squared prisms, of about $2\frac{1}{2}°$, $5°$, $10°$ and $15°$ respectively. Twice a day for ten minutes the patient was to fix a flame, or door knob, at 20 feet, and exercise divergence and convergence, beginning with the weakest prism, and mounting to the strongest.

Since then, numerous methods of exercise have been suggested by different writers.

I never now prescribe square prisms since patients rarely hold them correctly, but find it

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* *Journ. Anat. and Phys.*, vol. xxi., p. 21.

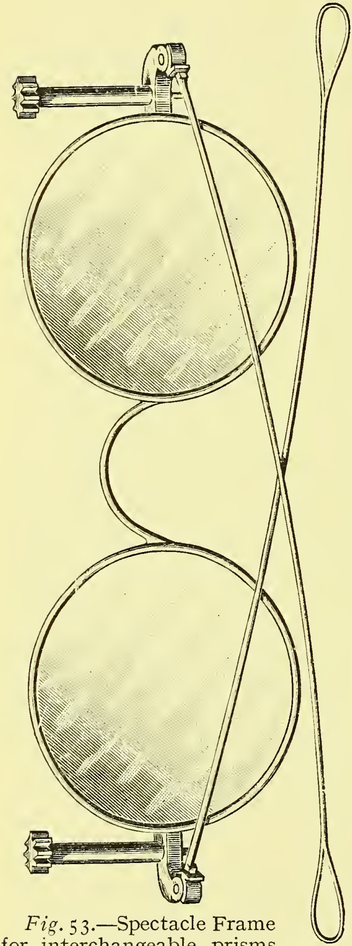

Fig. 53.—Spectacle Frame for interchangeable prisms (Culver) suitable for author's rotation plan.

better to order prisms mounted in ordinary circular-eyed spectacle frames. Instead of replacing these by stronger prisms as the exercises become able to be made more severe, the better plan may be recommended of prescribing at first two rather strong prisms with their apices pointing upward, but slightly divergent or convergent, so as to afford the resultant prismatic effect required for the first gentle beginnings of the treatment. On each visit of the patient the angle between the apices is increased by slightly rotating the prisms in opposite directions and fixing them again.

This expedient is available for the exercise of both vertical and horizontal deviations. For horizontal deviations such as exophoria and esophoria, the apices should, to start with, be nearly vertical, but each placed a little inwards for exophoria and a little outwards for esophoria, taking care that before finally fixing them the amount of inclination is the same for each prism. To ensure this it is well to look through them at a horizontal line on the wall, and see that the apparently raised portions of the line lie at the same level. A better plan still is to employ images by internal reflection to discover the exact direction of the apex while the spectacle frame is placed on an axis-finder for testing the axes of cylindrical lenses.

To exercise the relative vertical motions of the eyes, as for hyperphoria, both apices should, to begin with, point nearly to the patient's left, but one a little above, and the other a little below, the horizontal line. For right hyperphoria the apex of the right prism should be depressed a little, and that of the left prism elevated to an equal amount; the converse being adopted for left hyperphoria.

How is the patient to use these fixed exercising prisms? At first intermittently, by looking for brief intervals at distant objects, or, if that be found difficult, approaching the objects first, and then receding from them with the spectacles on until diplopia threatens. When the eyes have

learned to conquer this diplopia, the spectacles should be worn more permanently, say for half an hour at a time whenever the nervous conditions are strongest, as after breakfast or after dinner. In some cases the prisms can be worn almost all day until the heterophoria is finally cured.

Quicker results still are obtainable by the prism-verger used as follows:—

To practise supraverging exercises for hyperphoria we should start with both apices to the left, accomplishing this by reversing the position of the detachable prism if required. The rotation of the milled head will then make one apex rise and the other fall at exactly proportionate rates. The horizontal deviation of the visual lines induced by each prism is easily overcome by the patient slightly rotating his head to the other side, this being an easy and unconscious action. The vertical elements therefore remain.

For right hyperphoria the rotation should be effected in a direction which depresses the apex of the right prism, while for left hyperphoria it is the apex of the left prism that should be depressed, that of the right prism, of course, rising simultaneously. The trial frame is marked in such a way as to tell the amount of exercise attained, and the advantage of this apparatus over the fixed prism frame is that the patient is able to exercise his eyes more strongly on occasions when his physical power is greater.

CHAPTER XII

RELIEVING PRISMS

Orthoptic exercises are not always practicable. Many patients cannot spare the time for them, others are wanting in the energy and application required. In old people for instance, training prisms are unsuitable, and in their case especially a small or threatened diplopia needs immediate relief, since the giddiness which it may occasion is dangerous. Where training prisms cease to be of value, relieving prisms come to the front. They possess, however, the disadvantage of being only palliative, inasmuch as they do not contribute in the smallest degree to the cure of the condition. In many cases of commencing diplopia they afford great comfort, but before prescribing them it is always well to make sure that the diplopia is not of paralytic origin. This can be done by measuring it with the head placed in different positions. Should the diplopia be greater on looking in one direction than in another it has a paretic element, and prisms, though still perhaps valuable, will not be found so entirely satisfactory as when the diplopia remains unchanged in all parts of the motor field.

In addition to their use for diplopia, prisms are valuable for the large group of cases classified under the name heterophoria, and more particularly in hyperphoria. Let us consider in order :—

1.—**The Relief of Esophoria.** We need first to exclude causes which can be relieved by constitutional treatment—reflex, hysterical, cerebral, etc. Above all, hypermetropia needs correction.

If the esophoria does not exist for near vision, care must be taken that the prisms ordered should be worn only for distant vision. If it be found to exist in near vision also, they can be worn for all purposes. In myopia it is not uncommon to find considerable esophoria in distant vision giving rise to occasional diplopia, which can be remedied by displacing the lenses nearer together till the excess of convergence, shown by the rod test, is reduced almost to nil. It is well not to correct more than two-thirds of the esophoria.* Slight esophoria can be left alone.

2.—**The Relief of Exophoria in general.** Exophoria is a much commoner condition than esophoria and can generally be left alone. It occasionally, however, gives rise to diplopia, or to asthenopia and headache. It may then, if untrainable, be relieved by prisms which annul half the latent condition. As an alternative to the prescribing of prisms, convex lenses can be

* Noyes recommended its full correction, on his supposition that it is due to insufficiency of the externi. This is not a recommendation I feel able to endorse.

decentred inwards, and concave lenses outwards. Exophoria in distant vision is nearly always accompanied by still greater exophoria in near vision, so that in contrast with esophoria the same prisms can be worn for near work as for distant. In some neurasthenic individuals great variability occurs in the equilibrium of the eyes. These are not suitable cases for prisms.

3.—**The Relief of Exophoria at Reading Distance only.** In myopia, since accommodation is less than in emmetropia, the accommodative convergence is less. This is rectified by wearing the full correction for reading, but sometimes the full correction cannot be borne, and it is necessary to prescribe weaker lenses either displaced or decentred outwards, or combined with prisms, edge out. What frequently happens is that both distant and near exophoria exist, and then the correction of the former generally suffices if the same spectacles are used for all purposes. A high amount of exophoria at reading distance is often tolerated without discomfort. It sometimes occurs in cases other than myopic.

Illustrative case.—Mrs. L., 38 years of age, has had extensive corneal nebulæ from childhood. RV = $\frac{5}{60}$: LV = $\frac{5}{36}$. Equilibrium by rod test for distance, and the near vision tangent scale,

$$5 \text{ m.} = -1^\circ$$
$$\tfrac{1}{4} \text{ m.} = 12^\circ$$

She is obliged to hold her sewing, or small print,

near her eyes, to obtain sufficient visual acuity, yet this causes headache and asthenopia. As shown above, she has but slight exophoria for distance, namely, 1°, which is $\frac{1}{2}$° for each eye, and can be neglected. At $\frac{1}{4}$ metre she has 12° of latent divergence, which is 6° for each eye. She was ordered, for fine work, spectacles of +3·5 D, with prisms of 6° *d*, base in. With these she can hold her work near, and get good acuity without discomfort; the nebulæ making her less observant of the prismatic aberrations. She could not tolerate them at the first trial, but in a day or two said, "I would not be without them," and writes after a few months: "Spectacles very useful to me in reading."

4.—**The Relief of Hyperphoria.** This is one of the chief services rendered by prisms.

Well-marked cases of persistent hyperphoria, with discomfort from threatened diplopia and eye strain, are immensely relieved by prisms, which should correct, in my judgment, about two-thirds of the defect. The author has shown that prisms set vertically have less effect over hyperphoria in near than in distant vision, as illustrated on p. 178, so that full correction for distance is under-correction for near. The prisms should be equally divided between the two eyes, the edge of each being in the same direction as the displacement of the cornea, that is, edge up before the higher eye, and edge down before the lower.

Illustrative case.—Miss Mary T—, age about 45, with ·5D of hypermetropia in each eye and V = $\frac{5}{6}$, complains that she has had a tendency to double vision for the last two years, and that when she does see double, the objects appear superposed; thus, she saw two rows of cows, one above the other. A similar difficulty, encountered in reading, she had learned to overcome by tilting her reading spectacles to make one lens higher than the other (an unconscious use of prisms). The rod test shows at 5 m. right hyperphoria (i.e., the right eye highest) of 1°. She also complains of headaches and asthenopia, such that, though highly intellectual, she has almost ceased to read or use her eyes for close work for the last year or two. She remembers that, when a child, she could not see properly through a stereoscope, from the two images appearing superposed. This observation was of interest as showing it was a life-long condition; and repeated tests at considerable intervals gave precisely the same measurement. *Treatment.*—Prisms of $\frac{3}{8}°d$ before each eye, edge up before right, edge down before left, with lenses of + 0·5D. Similar prisms to be combined with her reading spectacles. *Result.*—The headaches greatly relieved; asthenopia gone; says glasses have taken her back several years in the use of her eyes, so that she can sew and read again as of old; but if she takes them off for a moment the diplopia is more troublesome than

before. This last fact illustrates the great disadvantage, already referred to, in the use of prisms prescribed for relief rather than cure. The prisms, being less than the hyperphoria, cannot really have created an increase, but have possibly allowed more of what was latent to become manifest. This, however, is more than compensated for by the relief they have given.

5.—**To correct Diplopia from Oculomotor Paresis.** In recent paralytic cases we, of course, wait for cure of the condition, but if it seem to come to a standstill, we may afford relief by prisms which enable the images to be united in at least the most useful direction of vision. At first, the diplopia occurs only in a certain area of the field of fixation, but after a while, through what is often, but wrongly, called "contracture of the antagonist," heterophoria spreads over the whole field. This heterophoria in the non-diplopic area should be corrected, and a little of the manifest diplopia as well. A prism should have its edge placed in the direction in which the cornea of the affected eye deviates, taking care, if we wish to divide the prism between the two eyes, that the one before the sound eye should have its edge in exactly the opposite direction. If, for instance, the right eye deviate down and out, its prism should be edge "down and out," and the prism for the left eye be edge "up and out." In old paralysis of the superior oblique, prisms are very

valuable, since, on looking down at the work, the diplopia is troublesome. The most thorough way to examine a case is to combine the vertical and horizontal scales of *Figs.* 44 and 47 with a flame at their centre, and then test the equilibrium of the eyes with the glass rod in each of the nine positions of the field of fixation; this is done by placing the patient's head in different positions. From a chart thus made, it is easy to reason out what prisms it is best to order. But a quicker way is just to place the patient's head so that on looking at the flame his axes of fixation are in the position mostly required at his work; the vertical and horizontal deviations can then be measured, and prisms be prescribed to correct a certain proportion of each. Thus, if the right eye deviate upwards 5°, we may order a prism of 2° *d*, edge up, before the right eye, and a like one, edge down, before the left. If the same eye also converges 3°, we may combine with each of the above a plus prism* of 1° *d*, in the mode explained on p. 64. A man with paralysis of the right superior oblique, who could look down at his work comfortably only by shutting one eye to avoid diplopia, greatly valued resultant prisms of 3° ($1\frac{1}{2}$° *d*) each, which were ordered him.

* I find it convenient to designate prisms set with their edges in, plus prisms, and with their edges out, minus prisms, and to record convergent deviations of the eyes as plus, and divergent as minus.

6.—**Cosmetic Prisms.** The cosmetic use of prisms was brought to my notice by a case in the practice of Mr. Berry, who succeeded in concealing, to a considerable degree, a vertical squint in a young woman, by prescribing a vertical prism to be worn before it, in conjunction with a lens equal to that needed before the seeing eye. Since prisms cause apparent displacement of objects towards their edges, the edge of a cosmetic prism should be in the opposite direction to the squint. Thus, if the eye squint up, the edge of the prism should be down ; if the eye squint out, the edge should be in. This procedure is chiefly applicable to slight vertical squints, which are more disfiguring than horizontal ones.

CHAPTER XIII

DECENTRING OF LENSES

INSTEAD of imparting sphericity to the surfaces of a prism where it is wished to combine a prism with a lens, the same result is attainable more accurately by simply decentring the lens, a process which, though already well known, may be simply explained.

The "optical centre" of a lens is the point traversed by all rays which, after entering the lens, emerge from it without change of direction. Each ray which does *not* traverse the optical centre is bent from its former course, and the more so the greater its distance from the centre. In other words, a lens, in every part other than the optical centre, has a deviating effect upon any single ray of light—the effect of a weak prism near the optical centre, and of a stronger and yet stronger one with every further removal from it; and this equally with concave and with convex lenses. It is evident, therefore, that by shifting a lens so as to bring a more peripheral part into use, a prismatic effect can be obtained just as much as if a prism had been combined with the lens in its former unshifted position. We can

alter the position of the optical centre in one of two ways, the prismatic equivalent of which is identical, but each of which possesses an advantage of its own. First, the lens may simply be *displaced*

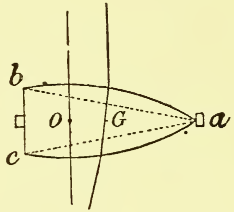

Fig. 54. A decentred lens.

as it is, rim and all, by altering the breadth of the spectacle front; or, secondly, the frame remaining unchanged, the lens can be *decentred* in its rim, as shown in *Fig.* 54. The focal length of this lens is exactly the same as of that shown

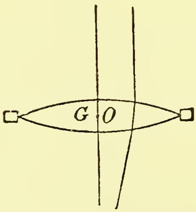

Fig. 55.—A lens normally centred.

in *Fig.* 55, inasmuch as its surfaces have the same radius of curvature, but it is as though it were cut peripherally, as shown in *Fig.* 56, *a b* from a larger lens *c d*, so that its optical (*O*) and

geometrical (G) centres do not coincide as in an ordinary lens. A brief study of *Fig.* 54, which gives the lens *a b*, of *Fig.* 56 in section, will show that the effect of cutting it eccentrically out of a larger lens is exactly the same as if a smaller lens

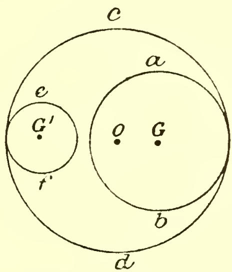

Fig. 56.—Peripheric lenses cut out of a larger one.

of the same strength had been split to admit a prism between its two halves. Without altering its focal length then, a prism has been virtually introduced. The *geometrical* centre of a lens is the mid-point of its diameters, as G in *Figs.* 54, 55, and 56. The *optical* centre, on the other hand, lies in the thickest part of a convex lens and the thinnest part of a concave one. For determining its position in any particular lens, methods will be described further on. A lens is said to be truly centred when the optical and geometrical centres coincide as in *Fig.* 55; but to be decentred when, as in *Fig.* 54, the optical centre (O) and the geometrical centre (G) are apart, the amount of decentration depending on their distance apart.

The two points may be so far removed from each other that the optical centre may come to lie outside the lens altogether. Such would evidently be the case with the small lens (*e f*) in *Fig.* 56, since the optical centre is at *O*. *Fig.* 57 shows this small lens in section, and it is evident at a glance that the two curvatures have nowhere parallel tangents, and that one or both of the surfaces would need to be prolonged considerably before any point could be found in the one having a tangent parallel to that of a point in the other; were this done, a line uniting the points would pass through the optical centre (*O*).

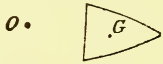

Fig. 57.—A lens with its optical centre (*O*) far outside itself.

We have seen that the effect of decentring a biconvex lens is equivalent to splitting it into two halves, and inserting a prism between them (*b a c* in *Fig.* 54). The strength of this (virtually interpolated) prism depends on, first, the amount of decentration, and, second, the focal length of the lens. A strong lens needs decentring to a less extent than a weak one to produce the same effect. Table V, (p. 123) not only illustrates this, but enables us to find the exact prismatic equivalent of decentring any lens. All that is required is to fix on the number of millimetres (in the highest horizontal row of figures) by which a lens

is known to be decentred, and the prismatic equivalent in degrees of deviation is found in the column beneath, opposite the dioptric strength of the lens. Conversely, should it be required to know how much to decentre a given lens in order to combine with it a certain prism, the eye runs along the horizontal line of angles opposite the known strength of lens till that angle is found nearest to the strength of prism required, when the number of mm. at the head of the column will indicate the decentring necessary.*

The word "decentring" is generally applied to making the optical centre no longer coincide with the geometrical centre of the lens, but we have already said that the prismatic effect is the same if the whole lens, rim and all, be simply *displaced*, as by lengthening or shortening the spectacle frame; in other words, it makes no prismatic difference, so long as the optical centre is moved, whether the geometrical one remains *in statu quo*, or whether it is shifted equally itself. How are we then to decide between these rival modes, in any given case? Decentring of the lens has the disadvantage of being attended with proportionate increase of weight, as much as if a prism were inserted: the lens of *Fig.* 54 is very evidently heavier than that of *Fig.* 55.

*Since this table was constructed many have been made by others with variations of several kinds, each valuable in its way.

This fault is entirely avoided by the alternative of displacing the entire lens (instead of only decentring the optical centre *in* the lens), but, on the other hand, such displacing entails, beyond certain limits, an unpleasant appearance and, worse still, a limitation of the field of binocular fixation. It may therefore be laid down as a practical rule, that the dislocation of the optical centre should be effected only slightly by displacing the whole lens, rim and all, leaving the remainder (should any more be required) to be obtained by decentration proper, that is, by decentring the lens *in* its rim.

Since the first edition, Mr. Prentice has introduced a simple and beautiful way of decentring lenses, the principle of which the reader will most readily understand if he take a strong convex and concave lens of equal power from the trial case. Viewing through them the scale of *Fig.* 17, or that of *Fig.* 44 with a vertical line dependent from its zero, no deflection of the zero line will be noticed while the optical centres are together. But if one lens be slid horizontally over the other, the index line will be seen to move with the concave lens. If, however, the optical centre of either lens be marked with an ink dot, and the sliding be regulated so that the index line shall point to that graduation of the scale which indicates the required prism, the lenses thereupon being firmly gripped so as not to disarrange their

relationship, the ink dot indicates the position of the required geometrical centre of the undotted lens. Given, therefore, any lens or sphero-prism (to have a required prismatic effect) we need only find from the trial case its neutralizing lens, and mark the optical centre of this neutralizing lens with an ink dot in order to find in the way described the proper geometrical centre of the rim or eye-wire.

As Prentice truly says, it would be well for manufacturers to provide themselves with "a series of lenses capable of a decentration of at least one centimetre. Such lenses would not require to be larger than six centimetres in diameter, and could be confined to the weaker powers, say from 0·25 to 2 dioptres. The cost of such lenses should certainly not be greater than that of sphero-prisms."

To take an example, let us suppose it desirable to prescribe spectacles such that each optical centre shall be 10 mm. nearer to the median plane than the centre of the eye, and that we have found the centre of each eye to be 32 mm. from the median plane. The effect of limitation of the field of binocular fixation must be considered, since it may cause inconvenience, and this will depend largely on the nature of the patient's occupation. Then, supposing that 5 mm. of *displacing* for each lens to be the maximum possible, this would make each geometrical

centre 27 mm. from the median plane, and leave 5 mm. for *decentring.*

In prescribing the spectacles, therefore, some such directions as the following might be given :

Geometrical centres 54 *mm. apart.*

Decentration of each lens 5 *mm. inwards.*

The optician would now have the choice of making these lenses by *pure decentration*, or by imparting sphericity to the surfaces of a *prism*, so as to obtain an approximate or coarse result, and then decentring this slightly to obtain the fine adjustment.

By the first method he would grind a large lens, of the focal length ordered, find its optical centre, and mark it with a dot, then make another dot 5 mm. distant and cut out the lens, with the last dot for its geometrical centre, placing it in a frame made with the centre of the rim 27 mm. from the middle of the bridge, so that the first dot (indicating the optical centre) shall lie to the inner side of the second dot.

By Table V. the strength of prism required for a proper filling of the above prescription is easily found, for opposite 4D, and under the column 5 mm. we find 1° 10′. This is the deviating angle of the prism, and needs nearly doubling for the physical angle (p. 23).

After selecting, therefore, the prism out of stock which happens to agree most nearly with this requirement (say $2\frac{1}{4}°$ of edge angle), and imparting the prescribed sphericity to each of

	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	9 mm.	10 mm.	11 mm.	12 mm.	13 mm.	14 mm.	15 mm.	16 mm.	18 mm.	20 mm.	22 mm.	24 mm.	26 mm.	28 mm.	30 mm.	32 mm.
·5D	1′ 43″	3′ 26″	5′	7′	8′ 25″	10′	12′	14′	15′ 28″	17′	19′	0′ 37″	22′ 27″	24′	26′	27° 30′	31′	35′	33′	41′	45′	48′	52′	55′
·75D	2′ 30″	5′	8′	10′	13′	15′ 28″	18′	20′ 37″	23′	26′	28′	31′	33′	36′	38′	41′	46′	52′	57′	1° 2′	1° 6′	1° 12′	1° 17′	1° 22′
1D	3′ 26″	7′	10′	14′	17′	20′ 37″	24′	27′ 30″	31′	35′	38′	41′	45′	48′	52′	55′	1° 2′	1° 9′	1° 16′	1° 23	1° 29′	1° 36′	1° 43′	1° 50′
1·5D	5′	10′	15′ 28″	20′ 37″	26′	31′	36′	41′	46′	52′	57′	1° 2′	1° 6′	1° 12′	1° 17′	1° 22′	1° 32′	1° 43′	1° 53′	2° 3′	2° 13′	2° 24′	2° 34′	2° 44′
2D	7′	14′	20′ 37″	27′ 30″	35′	41′	48′	55′	1° 2′	1° 9′	1° 16′	1° 23′	1° 29′	1° 36′	1° 43′	1° 50′	2° 4′	2° 18′	2° 32′	2° 46′	2° 58′	3° 12′	3° 26′	3° 40′
3D	10′	20′ 34″	31′	41′	52′	1° 2′	1° 12′	1° 22′	1° 32′	1° 43′	1° 53′	2° 3′	2° 13′	2° 24′	2° 34′	2° 44′	3° 6′	3° 26′	3° 46′	4° 7′	4° 27′	4° 48′	5° 9′	5° 29′
4D	14′	27′ 30″	41′	55′	1° 10′	1° 22′	1° 36′	1° 50′	2° 4′	2° 18′	2° 32′	2° 46′	2° 58′	3° 12′	3° 26′	3° 40′	4° 8′	4° 35′	5° 2′	5° 29′	5° 56′	6° 24′	6° 50′	7° 18′
5D	17′	35′	52′	1° 9′	1° 26′	1° 43′	2°	2° 18′	2° 35′	2° 52′	3° 9′	3° 26′	3° 43′	4° 1′	4° 17′	4° 35′	5° 9′	5° 44′	6° 17′	6° 51′	7° 24′	7° 58′	8° 32′	9° 6′
6D	20′ 37″	41′	1° 2′	1° 23′	1° 43′	2° 4′	2° 24′	2° 45′	3° 5′	3° 26′	3° 46′	4° 7′	4° 27′	4° 48′	5° 9′	5° 29′	6° 10,	6° 51′	7° 31′	8° 12′	8° 52′	9° 32′	10° 12′	10° 52′
7D	24′	48′	1° 12′	1° 36′	2°	2° 24′	2° 48′	3° 12′	3° 37′	4° 1′	4° 24′	4° 48′	5° 12′	5° 36′	6°	6° 24′	7° 11′	7° 58′	8° 45′	9° 32′	10° 19′	11° 5′	11° 52′	12° 38′
8D	28′	56′	1° 22′	1° 50′	2° 18′	2° 46′	3° 12′	3° 40′	4° 8′	4° 35′	5° 2′	5° 29′	5° 56′	6° 24′	6° 50′	7° 18′	8° 12′	9° 6′	9° 59′	10° 52′	11° 45′	12° 38′	13° 30′	14° 22′
9D	31′	1° 2′	1° 32′	2° 4′	2° 35′	3° 6′	3° 37′	4° 8′	4° 38′	5° 9′	5° 39′	6° 10′	6° 40′	7° 11′	7° 41′	8° 12′	9° 12′	10° 12′	11° 12′	12° 11′	13° 10′	14° 9′	15° 7′	16° 4′
10D	35′	1° 9′	1° 43′	2° 18′	2° 52′	3° 26′	4° 1′	4° 35′	5° 9′	5° 44′	6° 17′	6° 51′	7° 24′	7° 58′	8° 32′	9° 6′	10° 12′	11° 19′	12° 25′	13° 30′	14° 34′	15° 39′	16° 42′	17° 45′
12D	41′	1° 23′	2° 4′	2° 45′	3° 26′	4° 7′	4° 48′	5° 29′	6° 10′	6° 51′	7° 31′	8° 12′	8° 52′	9° 32′	10° 12′	10° 52′	12° 11′	13° 30′	14° 47′	16° 4′	17° 20′	18° 34′	19° 48′	21°
14D	48′	1° 36′	2° 24′	3° 12′	4° 1′	4° 48′	5° 36′	6° 24′	7° 11′	7° 58′	8° 45′	9° 32′	10° 19′	11° 5′	11° 52′	12° 38′	14° 9′	15° 39′	17° 7′	18° 34′	20°	21° 24′	22° 47′	24° 8′
16D	56′	1° 50′	2° 46′	3° 40′	4° 35′	5° 29′	6° 23′	7° 19′	8° 12′	9° 6′	9° 59′	10° 52′	11° 45′	12° 38′	13° 30′	14° 22′	16° 4′	17° 45′	19° 24′	21°	22° 7′	24° 8′	25° 38′	27° 7′
18D	1° 2′	2° 4′	3° 6′	4° 8′	5° 9′	6° 10′	7° 11′	8° 12′	9° 12′	10° 12′	11° 12′	12° 11′	13° 10′	14° 9′	15° 7′	16° 4′	17° 57′	19° 48′	21° 36′	23° 22′	25° 5′	26° 45′	28° 22′	29° 57′
20D	1° 9′	2° 18′	3° 26′	4° 35′	5° 44′	6° 51′	7° 58′	9° 6′	10° 12′	11° 19′	12° 25′	13° 30′	14° 34′	15° 39′	16° 42′	17° 45′	19° 48′	21° 48′	23° 45′	25° 38′	27° 29′	29° 15′	30° 58′	32° 37′

TABLE V.* Showing the prismatic effect of decentring any lens.

Procedure.—Find the dioptric strength of the lens in the left-hand column, the deviating angle of the virtual prism is then found opposite, under the number of mm. of decentring.

* This table is adapted from a thesis in 1884, where it appeared under another title (*Journal of Anat. and Phys.*, vol. xxi., p. 32). It was made from the formula: Tan. $x = \frac{d}{f}$

x, being the deviating angle of the virtual prism; d, the decentring in millimetres; f, the focal length of the lens in millimetres.

its surfaces, we should proceed to find the optical centre by one of the methods so well known, marking it with a dot. Now measuring 5 mm. from it in the direction of the apex of the prism, we should there make a second dot to stand for the geometrical centre of the completed lens, and would make the line connecting the dots agree with the long axis of the oval. In this way any inaccuracy in the selection of the prism is completely corrected. When weak prisms are combined with lenses the whole problem of accurate manufacture would be solved by this plan, or by Prentice's later one (p. 121) which is equally simple.*

With lenses too weak, or prisms too strong, for the optical centre to lie in the combination at all, the prism-measure of the Geneva Optical Company comes to our aid. Placing the selected prism, after grinding its faces to the right dioptric power for the coarse adjustment, under the teeth connected with the index, and with its base-apex line parallel to them, it is easy to find the position which makes the index point to the strength of prism which the table has shown us to answer to the 5 mm. of decentring required. The central tooth then points to the future centre,

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*In this country decentred lenses are not very popular with some manufacturers, since, curiously enough, they are not so well paid for them as for prisms, but there should be no difference, since careful work is so necessary to avoid (shall we call it?) miscentring.

which can be dotted with ink. As a little added precaution, the lens should now be rotated 90° to make sure that when the tooth points to the dot the index is at zero. This is to guard against the introduction of any *vertical* prismatic effect. If it does not point to zero, move the prismosphere laterally, and with care, till it does, when the new point marked out by the central tooth gives the true centre required.

When plain prisms are ordered, uncombined with lenses, we lose the power of adjusting their strength by any means of this kind.

(Should a workman prefer to work by a simple rule of thumb, rather than by a formula, the following is suggested :—

For each *per cent* of deviation, a lens of one dioptre must be decentred one centimetre.

For each *degree* of deviation, a lens of one dioptre must be decentred eleven sixteenths of an inch, or one and three-quarter centimetres.

To find the decentring in centimetres to give effect in centunes divide the number of centunes by the number of dioptres in the lens.

To find the decentring in sixteenths of an inch to produce a given deviation in degrees, divide eleven by the number of dioptres in the lens and multiply this by the degrees of deviation required. Decentring $= \frac{11 \times {}^\circ d}{D}$. Should it be the edge-angle of the prism that is given instead of the angle of deviation, read the

decentring in thirty-seconds instead of in sixteenths.*)

The formulæ of course are very simple: letting $°d$ stand for degrees of deviation; % for centunes (including centrads or prism-dioptres); and D for dioptres—

$$\text{Centimetres} = \frac{\%}{16}$$

$$\text{Inches} = \frac{11}{D} \times \frac{°d}{D}$$

In the case of cylindrical lenses, the effect of decentring depends on its direction. The effect of horizontally decentring a cylinder is greatest when the axis is vertical, and nothing when it is horizontal. With an oblique axis we need to find the focal length of the lens in the meridian of decentring. Here, though calculation would be easy, the lens-measure of the Geneva Optical Company is a useful alternative. By pressing it against the meridian under investigation, it records its dioptric strength at once. Adding this to the strength of the spherical surface, if the latter be of the same sign (or subtracting one from the other if the signs be different), we then proceed as before.

* For rapid mental work with degrees the following rule will be found to work most easily. To find the decentring in inches or centimetres divide 3″, or 7 cm., respectively by four times the number of dioptres in the lens. This gives the decentring required for each degree.

‡ To find the strength in dioptres, of the horizontal meridian of an oblique cylindrical lens, we need only, I believe, multiply the strength of the lens in dioptres by the sine of the angle of rotation of its axis from the horizontal.

As regards the distance of the optical centre from the median plane of the head, we should take into account that for a lens to have no effect on convergence, it must, at reading distance, be displaced 2 or 3 mm. more inwards than for distant vision. This position therefore forms the zero reading position, by displacing the optical centre from which in either direction the visual axis is deflected.

In spectacles for distance it is, as a rule, best for the geometrical centre of each lens to lie directly in front of the centre of the pupil, but in spectacles for reading purposes their distance apart should be half a centimetre less.

To relieve deficient convergence, prisms should be set "edge out"; convex lenses should be displaced inwards, and concave lenses outwards.

To relieve excessive convergence, prisms should be "edge in," convex lenses displaced outwards, and concave lenses inwards.

"Orthoscopic spectacles," as described by Dr. Scheffler, have been so long known that they must not altogether escape mention. In these, accommodation and convergence are affected in equal amount. The two lenses have but one optical centre common to both, just as though they were cut from opposite edges of one large lens. The proof of their correct manufacture is that they throw a *single* image of a flame upon a wall at their focal distance. They have generally

been prescribed in a few standard strengths, but they can be made for any patient by combining with each of his lenses a prism, whose deviating angle is found in Table V., opposite the description of lens, and under the number of millimetres by which the actual centre of each eye is distant from the median plane. These spectacles undoubtedly may have their place, but it is better to ascertain the conditions of convergence in each case and prescribe accordingly, rather than to adhere to any fixed combinations.

CHAPTER XIV

PRISMATIC ABERRATION

With the exception of chromatic aberration (p. 29), which prisms share in common with lenses, the optical imperfections of prisms are chiefly due to the fact that instead of a constant ratio existing between the angles of incidence and refraction, it only exists between the sines of those angles (p. 196). We have seen on page 17 that parallel rays, entering a prism, also emerge parallel; but with this exception, all homocentric light, that is, all light proceeding from, or going towards, a single point, is, after traversing a prism, no longer homocentric. The stronger the prism, and the greater the departure of the beam of light from the direction of minimum deviation, and the more converging or diverging its component rays, the greater is the prismatic aberration. There are two kinds of prismatic aberration, which have not been sufficiently hitherto distinguished from each other. The first concerns the shape of each pencil of light which is incident on the prism and then fills the pupil; this belongs to the domain of astigmatism.

The second kind of aberration concerns not the shape of the pencils, but the unequal deviations of different pencils. This causes metamorphopsia and involves the problem of the visual angle.

Prismatic Astigmatism.—A cone of light, after having passed through a prism, is somewhat flattened, its section having become slightly elliptical instead of circular. Two of its bounding rays which enter the prism at equal distances from its edge have a more distant focus than two which enter it in a plane perpendicular to the edge.

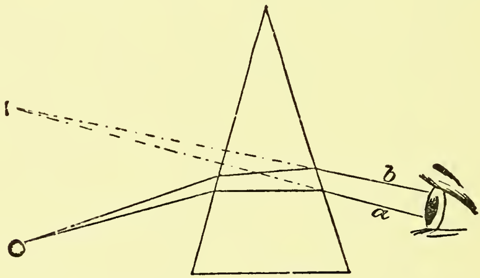

Fig. 58.

In *Fig.* 58, two rays proceeding from a point (*O*) enter the pupil of an eye in the vertical meridian. The inferior ray (*a*) is drawn so as to traverse the prism in the direction of minimum deviation, and it therefore experiences the least possible deflection with which any ray can pass through a prism.

Since the superior ray (*b*), having a different angle of incidence, cannot also suffer minimum deviation, it experiences a greater deflection, so that on their emergence the two rays are less divergent than they were before. On this account they enter the eye as if they came from *I*, a little farther off than if there were no prismatic astigmatism.

On the other hand, the bounding rays which proceed from *O*, and have an incidence equidistant from the edge of the prism, enter the eye as if they came from a point a little nearer than *I*. The interval between these two points is the focal interval.

In other words, every perpendicular band of light in the pencil has its virtual focus farther from the eye than every horizontal band.

Suppose now the prism to be tilted a little round its axis, so as to bring the superior ray, instead of the inferior, into the direction of minimum deviation. The conditions would then be reversed, for the inferior ray would be most deflected, and the two rays would enter the pupil more divergent, and as if they came from a point *nearer* than they would do if there were no prismatic astigmatism. We see, therefore, that the nature of the astigmatism is altered according as the apex of the prism is moved (away from the position of minimum deviation) towards the object, or away from the object. The greater,

too, the tilting, the greater the astigmatism.*

Prismatic Metamorphopsia.—A far more important consequence of prismatic aberration is that objects appear altered in shape when viewed through prisms. If square objects are looked at through a tilted prism, they appear oblong, being unaltered in the dimension parallel to the edge of the prism but narrowed at right angles thereto if the base of the prism be anterior to the apex, and broadened if the apex be anterior to the base. Hence, if a scale of equidistant figures on the wall be viewed through a prism with its apex to the right, the intervals between the right-hand figures will appear larger, and those between the left-hand figures smaller than they should be.

With the thin prisms employed in ophthalmology the error will be practically inappreciable if, instead of drawing the section of a prism in the usual way, we represent the prism by a straight line lying midway between its faces, and which,

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* What about the rays which enter the horizontal meridian of the pupil ? Even they are not simple in their behaviour, but the more peripheral ones traverse the prism higher (i.e., nearer the edge) than the central ones, so that they describe an increasingly curved surface as they proceed from *O* till they reach the prism, and after leaving it, this transverse curve (with its concavity upwards) becomes less and less, till they enter the pupil in the straight line of its horizontal diameter.

This will be easily understood after the explanation of the second form of prismatic aberration, since it depends on the fact that the peripheral rays have, after leaving the prism, a longer course than the central ones.

therefore, may be regarded as a section of its plane.* To identify this line, and indicate at the same time the direction of the edge of the prism which it stands for, let us draw a tiny prism at one end of it, as in *Fig.* 59. This figure illustrates both prismatic astigmatism and prismatic metamorphopsia. The first concerns the shape of each pencil. Rays proceeding from the

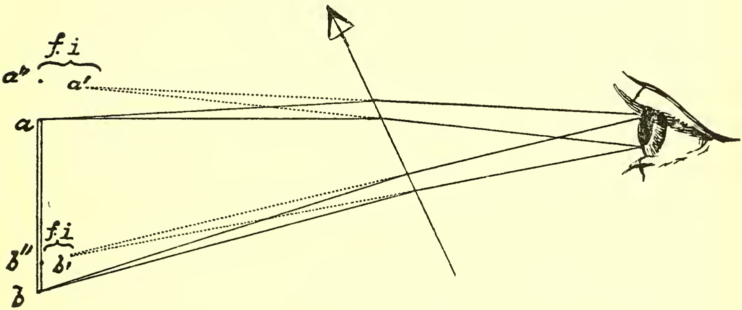

Fig. 59.—New figure to illustrate prismatic aberrations.

point (a) of the object (a b) enter the vertical meridian of the pupil as if they came from a', and the horizontal meridian as if from a''. The space between a' and a'' is the "focal interval" of Sturm, which represents the astigmatism of the pencil.

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* A straight line through the substance of a prism, parallel with the edge, and lying half-way between the edge and the base, is termed the *axis* of a prism, while the plane, common to the edge and the axis, and which therefore bisects the angle of the prism, and lies equidistant from its two plane faces, is called the *plane* of the prism.

For rays from the other end of the object (*b*) it will be seen that the focal interval is smaller, for the simple reason that this pencil traverses the prism less obliquely.

It will be noticed that the interval (*a′ b′*) is greater than the object (*a b*), so that the latter appears magnified in the vertical dimension, though in the horizontal dimension it remains unchanged. This metamorphopsia is due to the greater deflection of the pencil from *a* than of that from *b*. Were the prism to be tilted about its axis so as to bring its edge nearer the eye and its base nearer the object, the reverse would be the case. The pencil from *b* would have the most oblique incidence, so that *b′* would be raised more than *a′*, and the object would appear stunted vertically.

It is quite easy from a study of this diagram to see: (1) That a prism distorts objects only in the dimension that coincides with its base-apex line; (2) That when the base is tilted towards the eye, objects appear diminished vertically; (3) When the apex is tilted towards the eye, objects appear elongated vertically.

When a prism is rotated about its own base-apex line, another set of distortions appear, described first by Percival. Thus, e.g., when a prism, edge up, is held before the right eye the vertical piece of a cross on the wall appears to slant to the right when the right edge of the

prism is advanced, and to the left when the left edge is advanced. The horizontal arm of the cross becomes tilted, with its right end higher in the first case and lower in the second.

This distortion, therefore, closely resembles that produced by an obliquely set cylinder. From the nature of this distortion, it probably causes more trouble than any other when prisms are prescribed, since it throws embarrassing work on the oblique muscles whenever certain peripheral portions of the lens are looked through. It has occurred to me, indeed, that advantage could be taken of this property of prisms to exercise the oblique muscles in cases of excessive cyclophoria, by looking at vertical lines through two prisms with their edges up or down, and tilting them symmetrically about their vertical axes.

A hitherto unexplained form of prismatic aberration is observed when a long straight line is looked at through a prism. It appears curved, with its concavity upwards, if the apex of the prism is upwards. It is easy to give an explanation of this phenomenon even to an unmathematical reader. Take a paper triangle, as in *Fig.* 60, and draw a line bisecting its acute angle, such as *a f*. Then, if we imagine the base (*b c*) of the triangle to be the long horizontal line we are looking at, and *a* the position of the observer's eye, the lines *a b*, *a f*, and *a c* represent three rays of light as they would reach the eye were no prism

in the way. We may, for our purpose, suppose the size of the pupil to be inconsiderable, so that rays from every point of the line travel to the eye at *a* in one plane—the plane of the paper. Now bend the triangle at *d e* over the sharp edge of a table, and we have what at first sight we might think to be the alteration in the direction of the light to be introduced by interposing a prism, edge up, in the position *d e;* for the prism deflects

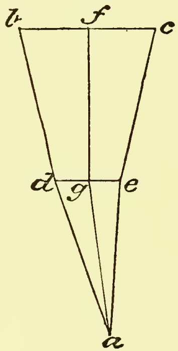

Fig. 60.

every ray of light towards its base, one as much as another.

The paper triangle does not, however, now represent the case, for if it did we should not see the line curved, but straight, since the light would still enter the eye in one plane, for the smaller triangle (*a d e*) is just as much a plane as the whole triangle was at first. What, then, is the explanation? It is this, that the distances *a d* and *a e*

being greater than the distance *a g*, the *gradient* is less, as even a horse knows, who prefers to ascend a hill obliquely. A fly walking down *e a* would not have so steep a walk as one walking down *g a*. The paper, therefore, does *not* represent the course of the rays through a prism, for a prism imparts equal gradients (that is, equal steepness) to all the rays. For the rays *d a* and *e a*, therefore, to possess the same gradient as *g a*, they must traverse the prism higher up, just as a long hill is higher than a short one of equal steepness. If we took a triangle of elastic material, and stiffened the base of the triangle, and then bent it at *d e* over a *curved edge*, with its concavity upwards, we would then represent the course of the rays of light, and show why the line appears curved.

So far, we have only spoken of prisms held before one eye, the other being closed. *Binocular distortion* results when both eyes are fitted with prisms, and has received its partial explanation, I believe, from Wadsworth, to whose paper* I would refer those interested, since it is difficult to give a just account of it without reproducing his diagrams in full. It depends on the different deviations produced with different angles of incidence, as already described in speaking of prismatic astigmatism, and accounts for the spherical appearance of flat surfaces when they are viewed through prisms.

* *Trans. Amer. Ophth. Soc.*, 1883.

As Wadsworth truly observes, since tangents increase faster than their angles, it follows that when we look with one eye at a plane surface, the details of that surface are more crowded together in the peripheral portions of the retinal image than in the central portions. But we have learned to disregard this, and consider it normal. Prisms with edges *out* disarrange this peripheral overcrowding, in binocular vision, in a way which reminds us of a spherical or cylindrical surface. Not only so, but the eyes, in glancing from detail to detail, make different excursions when viewing the periphery of a spherical surface than that of a plane surface, in proportion as the tangent to the surface is inclined to the visual axis. A similar alteration in the excursions, when we turn the eyes from side to side, takes place in consequence of wearing prisms. It is really the altering of the motions of the eyes which accounts most for the illusion. It is most marked when prisms are worn edge out; a plane surface then appears convex towards the observer. With prisms edge in, a plane surface appears concave.

The minimum deflection of light by a prism is invariable, but the degree of deflection of an eyeball before which it is placed depends on the use that is made of it. For instance, when a pair of converging prisms are worn, their effect on convergence is least on looking straight forward to a distance, and becomes greater on looking to either side, and also when used for near vision. This is due to the pencils of light traversing the prism more or less obliquely and

therefore in a direction more removed from that of minimum deviation. When diverging prisms are worn, the effect of the pair on convergence is also greater on looking to either side than on looking straight forward, and there is a certain distance, dependent on their strength, beyond and within which their effect increases. These variations, quite unimportant in the case of weak prisms, might be lessened by placing the bases of converging prisms slightly in advance of their apices, and the bases of diverging prisms slightly in advance for distant and further back for near vision, in the rare cases in which very strong prisms are ordered.

CHAPTER XV

MEASUREMENT OF THE INTEROCULAR DISTANCE, AND OF THE DISTANCE BETWEEN THE OPTICAL CENTRES OF SPECTACLE LENSES

It is useless to prescribe prisms or decentred lenses unless account be taken of the distance between the two eyes, and preferably between their visual axes during distant vision; for everything depends on the proportion which this distance bears to the distance between the optical centres of the two spectacle lenses.

A great many modes of measuring the interocular distance exist. The simplest and best known consists in so holding an ordinary graduated rule that its zero shall coincide with the left margin of the patient's right pupil as viewed by the observer's left eye; then the required distance can be read off by the left margin of the left pupil as viewed by the observer's right eye. This method, though long known in this country, does not appear to be universal, for some make the observation with one and the same eye, and add a certain amount (2 mm.) to compensate for the error which is thus

introduced. The simple but effective expedient of curling a strip of white paper into a loose gummed ring to slide on a metal rule and facilitate observation, is due to M'Gillivray, who also employs the same for measuring the distance between the optical centres of spectacles.

All who use graduated rules, however, are conscious of their disadvantage. The patient's head may move between the readings of the respective eyes, and besides this many of the rules provided throw a shadow over the eyes.

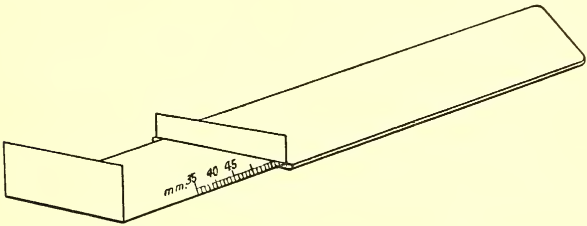

Fig. 61.—Perspective view of author's Inter-pupillometer.

The author has quite recently introduced an entirely new method of measuring the distance between the pupils, which enables both to be kept in view at the same moment, and apparently brought together after the manner of a sextant, but without the necessity, as in that instrument, of knowing the distance from it of the objects under observation. It consists simply of two mirrors facing and parallel to each other, but each inclined at 45° to the line of motion along which they are capable of separation. One

form of the instrument is shown in *Figs.* 61 and 62. In another form they are separable by means of a screw.

That shown in the figure consists of two strips of metal, each turned up at one end so as to support one mirror apiece. By sliding one strip over the other the distance between the mirrors can be adjusted with one hand. The observer can stand at any convenient distance from the patient, from a few inches to a yard or more, and while glancing at the patient's left eye just over the top of the shorter mirror he will notice

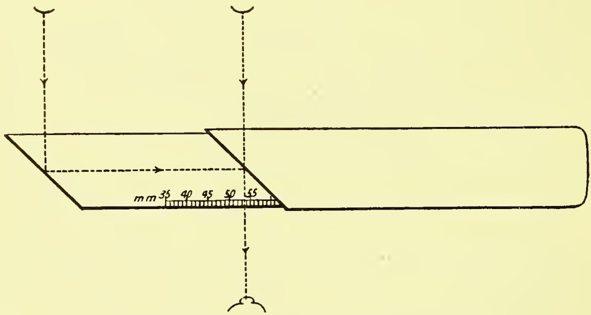

Fig. 62.—Plan of Inter-pupillometer.

a reflection of the patient's right eye in its neighbourhood. By adjusting the distance between the mirrors the two pupils are brought into one vertical line, or indeed with a little practice the lower half of the right pupil can be brought to fit against the upper half of the left.

To insure that the instrument is held strictly

at right angles to the line joining the observing and the observed eyes, each mirror is provided with a vertical mark; when these two marks are maintained in apparent coincidence the condition mentioned is strictly complied with.

Fig. 62 shows the instrument in plan. The dotted lines represent the course of the axial rays of light which enter the observing pupil from corresponding points of the patient's two pupils. The ray from the left eye travels direct, while that from the right eye undergoes double reflection.

The separation of the mirrors being linear, instead of angular as in the sextant, the distance from the patient does not affect the result, provided the instrument be made accurately.

Since a certain amount of light is lost at each reflection, it is necessary to keep the mirrors bright and clean.

In taking the measurement, a patient should be instructed to look into the distance over the observer's head. Should some bright, distant object, however, be available, its two corneal reflections can be brought into apparent coincidence by the instrument, instead of the pupils themselves.

The name "duant" might perhaps be given to instruments working on this principle, as keeping in mind its relation to the sextant.

So far we have only investigated the distance

between the two eyes, and not the distance of each from the median plane. When spectacle lenses are of equal strength, greater distance of one eye from the median plane than the other is immaterial so far as the joint prismatic effect is concerned, for though it is true that one lens will behave as if decentred more than the other, their combined effect on convergence will be almost the same as if no asymmetry existed, the greater prismatic effect of one lens being compensated for by the lesser prismatic effect of the other. When, however, the lenses are of highly disproportionate strength for the correction of anisometropia, this statement no longer holds good, and if we wish to know the exact prismatic effect we must ascertain the exact distance of each eye from the root of the nose. Some rules are supplied with a semicircular notch to rest on the nose while the measurement is taken, but estimates thus obtained are liable to considerable error from parallax, owing to the absence of any means of securing that the observer's visual line shall be at right angles to the patient's interocular line.

For a measurement to be true in principle, not only must the visual lines of the observing and observed eye coincide, but the line so composed must be exactly at right angles to the line uniting the centres of the two eyes. To overcome the difficulty a little device, *Fig.* 63,

may be used, with two sights so disposed as to be always in a line at right angles to any trial-frame in which it is placed. One pupil is localized at a time, all that is necessary being to increase or diminish the breadth of the

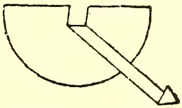

Fig. 63.—Localizer (from first edition).

frame till the two sights and the centre of the pupil are in line, as in *Fig.* 64; the patient meanwhile is bidden to be looking at the pupil of the surgeon's observing eye, by doing which the two visual lines (i.e., that of the observing

Fig. 64.—The Localizer in use.

and that of the observed eye) come to practically coincide. If the patient be a child it is quite easy to excite its interest in its own miniature reflection in the surgeon's pupil while taking the observation, adeptness in which needs a little practice to become familiar with the parallactic motion. The instrument is then placed in the

other rim of the frame, and the observation repeated for the other eye.

By this method we comply with the two afore-named conditions of accuracy ; the visual lines of the observing and observed eyes coincide, and of necessity they are both at right angles to the trial-frame. The frame must, of course, be one movable by a screw, and marked in millimetres, so that the record can be read off after localizing

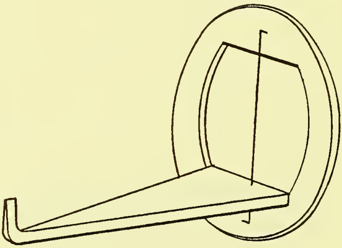

Fig. 65.—Localizer for Ophthalmoscopic Reflection. (Present edition).

each pupil. The distance of the centre of each eye from the middle line is in each case *half* what is registered by the frame. The careful use of this instrument will show how many an un-suspected difference exists in the position of the two eyes.

Though for most clinical work it suffices to know the positions of the pupils, it has been shown elsewhere that the corneal reflection of an ophthalmoscope, at the sight hole of which the patient is directed to gaze, affords the most

precise method of locating the visual axis. *Fig.* 65 shows a slightly different localizer adapted for making this measurement, by aid of the brilliant little star which appears on the cornea.

Space forbids the description of the numerous other methods of measuring the distance between the eyes, such as Smee's visuometer, Landolt's chiasmometer, Hess' visuometer, and Lucien Howe's telescope visuometer, in the last of which a micrometer scale is placed in the eye-piece of a telescope used at a measured distance.

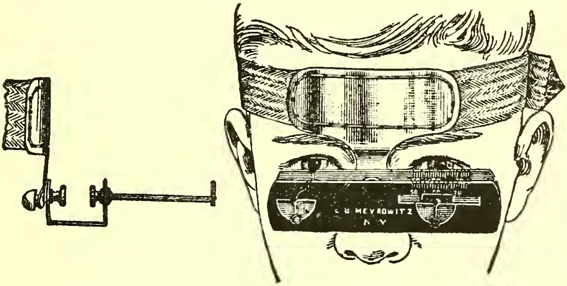

Fig. 66.—Visuometer of Dr. Lucien Howe.

Another instrument by Dr. Lucien Howe, shown in *Fig.* 66, deserves mention. It utilizes the principle of avoiding parallax, previously exemplified in the localizer, and is practically a double localizer. It, however, only measures the distance between the two eyes, and not that of each from the root of the nose.*

*"On the Measurement of the Interocular Base Line, and Size of the Metre-angle." *Prize Essay by Dr. Lucien Howe*, 1901.

Measurement between optical centres of spectacle lenses.—When a patient comes to us with a pair of spectacles already in use, how are we to investigate their prismatic effect, and find how the glasses are centred? Clinically, nothing is easier than to test the positions of equilibrium by the glass rod, etc., with and without the spectacles. We then see what effect they have on the visual axes, and by what distance lenses of similar focus, when placed in a trial-frame, need to be separated to produce a desirable equilibrium.

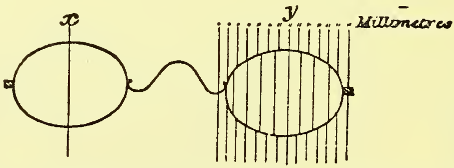

Fig. 67.—An imperfect mode of telling the distance between the optical centres of a pair of spectacles.

We can reach the same result in a surer way by measuring the distance between the centres of the eyes and between the optical centres of the lenses. If these two distances are equal, and the spectacles be for distant vision, they have practically no prismatic effect. With reading spectacles, the distance between the optical centres should, to avoid prismatic effect, be 5 or 6 mm. less than the interocular distance.

In *Fig.* 67 is represented a rough but ready method of telling the distance between the optical centres. The spectacles are held hori-

zontally, and some inches above a card marked to one side with a vertical line *x*, and towards the other side with parallel lines *y*, at different measured distances from the single one. The observer, holding his left eye about a foot above, and as nearly vertically over the left lens as he can judge, closes the right eye, and moves the spectacle frame from side to side till the line *x* appears unbroken by the left lens. Now, without moving the head, the right eye is opened and the left closed, and that line which appears to be least displaced by

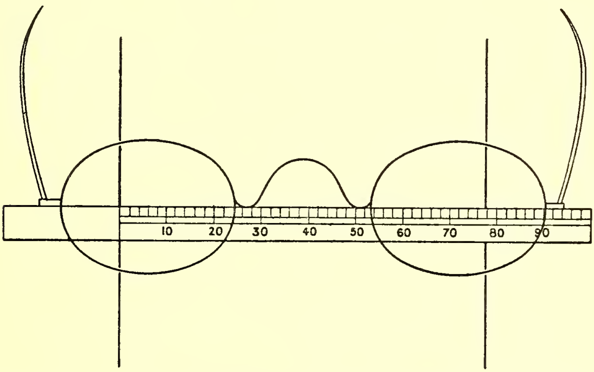

Fig. 68.—Author's narrow rule for measuring distance between the optical centres by M'Gillivray's ray's method.

the right lens is the line nearest its optical centre, so that the number above it furnishes a rough estimate of the required distance between the two optical centres.

This manœuvre is, however, a very poor one, being liable to so much error from parallax, and

a much better is illustrated in *Fig.* 68. A narrow strip of boxwood, square in section, is marked with one cross line about an inch and a half from one end, and with a graduated scale towards the other end. It is held against the spectacle lenses while some convenient vertical line, the distance of which depends on the strength of the lens, is regarded through that lens which is in apposition with the zero cross line of the ruler. After getting the vertical line to appear unbroken through the lens, the strip of boxwood is slipped sideways until its cross line appears to coincide with the unbroken vertical line, being thereafter held in rigid contact with the lenses. Its cross line indicates now the optical centre of one lens, and it only remains to find how far therefrom lies the optical centre of the other lens. This is done with equal ease, the same vertical line being regarded through it, while the frame is moved until the line appears unbroken. The unbroken line coincides with that figure on the scale which describes the distance between the optical centres. This method is M'Gillivray's without the paper (p. 141), but the narrow boxwood rule allows the distant line to be seen on both sides of it through the lens, and it grips the glass better than metal. A slight complication is introduced when the frame is glazed with strong oblique cylinders. The vertical line looked at cannot then be made to appear unbroken, owing

to the apparent obliquity of that part of it seen through the lens. *Fig.* 69 shows how this little difficulty is overcome. The lens is moved until the distant line as seen through the glass appears displaced as much in one direction above as in the other direction below. The midpoint of the tilted line then corresponds with the optical centre, and since the upper margin of the rule lies along the major axis of the ellipse it is just in the most favourable position for reading it off.

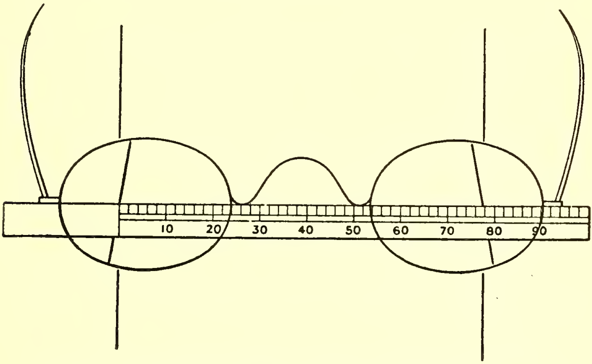

Fig 69.—Showing method with cylindrical lenses.

To ascertain the vertical decentration, the same distant line is employed, but the lens is held with its long axis vertical and the same rule horizontal. When that part of the line seen through the glass is made to appear continuous with the rest, it passes through the optical centre, the distance of which from either edge can easily be read off. It is equally easy to measure

the distance of each optical centre from the middle of the bridge.

If, however, the optical centre lie outside the lens altogether, then the "analyzer" in *Fig.* 70, which was described in the first edition

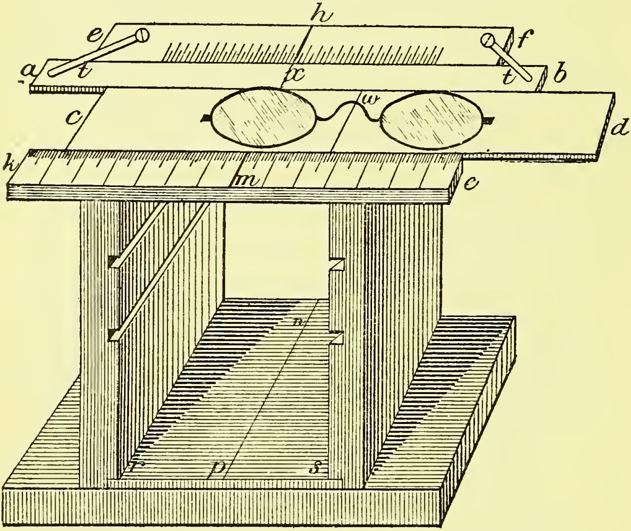

Fig. 70.—The Analyzer (from first edition).

of this work, can be used, though it has since been made unnecessary by the introduction of the "prism measure" of the Geneva Optical Company. The analyzer, though more difficult to use, possessed the advantage of giving a dioptric measurement true for media of any index of refraction, while the "prism measure" only gives the physical inclination of the surfaces. The prism

measure, shown in *Fig.* 71, is very simple in its use. If a lens be placed within it, and be moved about in the horizontal plane till the index points to zero, the tooth in the centre of the foot of the index points to the optical centre. If, on the

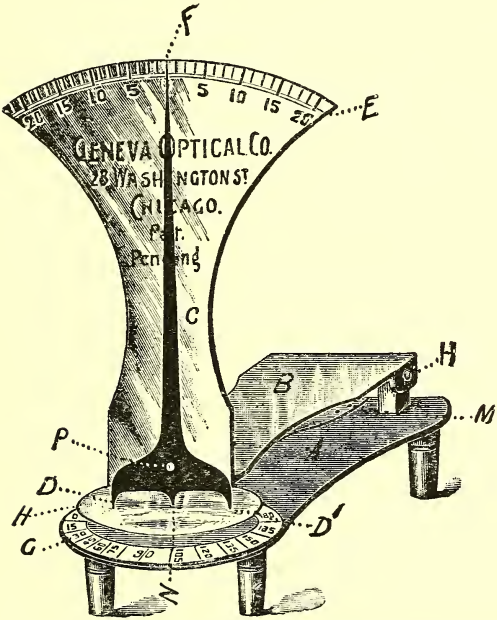

Fig. 71.—The "Prism Measure," used to discover the optical centre of a lens.

other hand, the geometrical centre be placed just below this tooth, the index points to the number which measures the physical angle of the virtual prism with which the lens is combined.

We may indeed go further, and find the

virtual prism in the line of the visual axis. The geometrical centre may not be directly in front of the eye, but if we measure from the centre of the bridge a distance equal to the distance of the centre of the eye from the median plane, and mark this distance by an ink dot on the glass, we can then place the said ink dot under the tooth and read off what prism is virtually combined with the lens at this point directly in front of the eye. This reveals the prismatic effect of the spectacles without any calculation.

CHAPTER XVI

PRISMS TO DETECT AND TO ELICIT BINOCULAR VISION

The use of Prisms for the Detection of Malingering.—So many excellent tests exist for the detection of simulated blindness of one eye that prisms only represent a small share of our resources.

1. Take a prism, preferably square, and with a sharply cut base; hold the base half way across the pupil of the eye acknowledged to be good, while the other eye is covered by the hand. Direct attention to a lighted candle and show how two images of it can be seen. A slight push of the prism further on, preceded by the removal of the hand from the feigning eye, completes the test. If the patient still sees two images, evidently one must be seen by the now uncovered eye.

2. While closely watching the feigning eye, place a prism with its edge in suddenly before the good eye. If the feigning eye instantly makes a tiny movement toward the temple, it is proved to be either blind, or to have vision inferior to the other. If it remains absolutely stationary, it

is a good eye, but let it not be forgotten that a tiny movement may easily be overlooked by the surgeon. So that a negative result possesses a much less value than a positive one.

3. Place a prism of more than 4° *d* edge out, before either eye and watch the other. If it make slight sudden movements from side to side, the patient sees double and is anxiously directing his attention to first one image and then the other, in order to decide which he should confess to if asked. If on placing this prism before, or removing it from, the feigning eye, either eye makes any movement, the sham is discovered.

Mr. Adams Frost has pointed out a possible but unlikely source of fallacy in making this test, namely, that the image by double internal reflection, explained on p. 48, may be noticed by the patient, who may thus say that he sees double, though the other eye be blind. The image spoken of is extremely faint, but though with a strong prism it is too far removed from the true image to attract attention, it might be noticed with a weak one.

To elicit Diplopia in cases of Suppression of the False Image.—After a squint has lasted a sufficient length of time, the mind has so learned to ignore the false image that it is impossible to produce diplopia by looking at a flame. The false image is then said to be "mentally suppressed." If the suppression be not deep,

attention can be called to the false image by placing a piece of green or red glass in front of the fixing eye, or, if this fail, by placing a vertical prism before either eye to throw the picture of the flame on to a different part of the retina. In alternating squint the false image is generally suppressed, whichever eye fixes. The suppression is sometimes so deep, in long-standing cases, that nothing can overcome it. In a few cases the glass rod has elicited diplopia when prisms have failed to do so: it should be combined with a glass of complementary tint before the other eye.

CHAPTER XVII

THE FUNCTION OF CONVERGENCE

According to Hering's well-proved theory, both eyes move together as though they were a single organ. No ocular muscle can in health receive a nervous impulse without the transmission of an equal impulse to some associated muscle of the other eye. Thus there is one conjugate innervation which turns both eyes to the right, and another which turns both to the left. Two others turn both eyes together upwards and downwards, several more adjust the vertical balance of the eyes and their wheel movements, as described in a companion volume. By these innervations, which need not detain us, the visual axes can be simultaneously moved in any direction, but were there no other innervation, we should see all near objects double, from inability to converge the visual axes upon them. This want is supplied by the innervation of convergence, which stimulates the internal recti equally, and independently of their control by the conjugate lateral innervations. To balance this converging innervation, there is most likely a diverging innervation. The efforts of

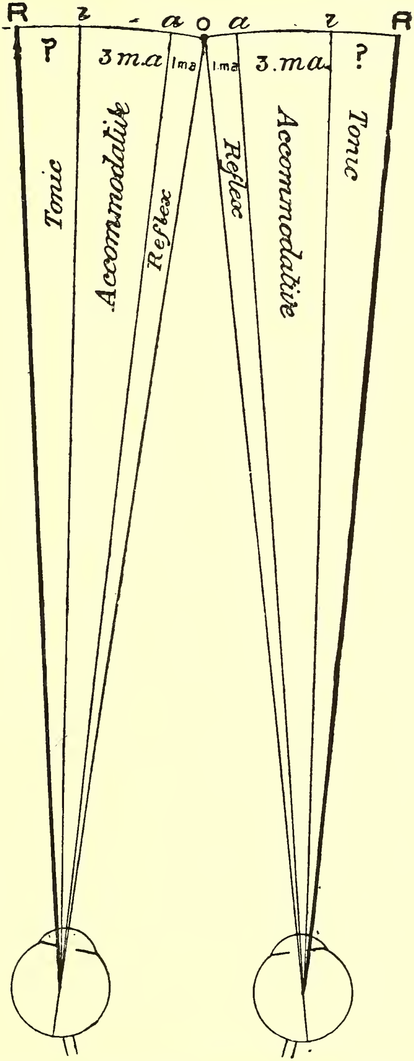

Fig. 72.—The three grades of convergence in vision for ·25m. To scale : half life size.

convergence and accommodation are intimately intersusceptible, so that the slightest impulse to either makes a difference in the other, albeit the difference is not necessarily equal in amount. If one eye be covered while the other is fixing a near object, the occluded eye will, in the majority of persons who are neither hypermetropic nor presbyopic, deviate outwards 3° or 4°. This behaviour of a normal eye, placed in the dark, was first noted by the use of the blind spot.* V. Graefe had long before shown how by placing a prism, edge up, before one eye, and looking at a dot on paper with a vertical line through it, he could detect in many cases of myopia "muscular insufficiency," as he named it. But his test did not appear to reveal that a lesser degree of the same thing was a frequent physiological condition. *Fig.* 45 (p. 79), shows a device for measuring this exophoria in near vision, the results of which agree with the blind spot method. It consists of a lithographed scale, graduated in tangents of degrees to right and left of a central zero, from which rises a vertical arrow. The scale is held ·25m. before the eyes, and is reduplicated by a square prism of 6° *d*, held vertically before one eye. The lower arrow then points upwards to that number which measures the deviation of

* "On a new method of determining the relation between Convergence and Accommodation of the Eyes."—*Trans. Ophth. Soc.*, Vol. iii., p. 290.

the eye. This divergence does not exist in ordinary vision, being overcome by a subtle visual-reflex action (fusion reflex) which protects us from seeing double, and which is discontinued when the false image produced by a prism is so far removed from the true one as to make it hopeless to attempt their union by overcoming the prism (*cf.* p. 57). The exophoria revealed by this test shows that the association of convergence with accommodation is not complete centrally, so that a supplementary action is needed to supply the deficiency, and, since the involuntary motive for this action is afforded by the desire to fuse two images into one, the proportion of convergence thus kept up may be called the "fusion convergence."

In ordinary binocular vision, when neither eye is covered and both work together, there is perfect concert between accommodation and convergence, so that the two visual axes meet exactly at whatever point is accommodated for. The fusion function is maintained in activity by the desire of the cerebral centre to preserve the union of the two images. How fine is the mechanism at work may be realized when we consider that if double images are produced artificially, or by disease, it is impossible for *the conscious mind* to tell to which eye each image belongs—whether, therefore, the visual axes are crossed or not, and whether convergence

needs to be increased or relaxed to bring the images together.

An action so complex must necessarily be more tiring than the mere overflow of one impulse into another. We should expect therefore the "fusion convergence" to involve a greater waste of co-ordinating nervous energy than the "accommodative convergence" which is associated with the act of accommodation. In many cases this waste is of no consequence, but in others it may give rise to the so-called "muscular asthenopia" of Von Graefe, which, however, is really in many instances a *central asthenopia*, though there may sometimes be a muscular element as well. Asthenopia from this cause can be relieved by prisms with their edges out, which lessen the convergence necessary for single vision. It is of course desirable to only relieve a sufficient portion of the "fusion convergence," and not the whole of it. Suppose an asthenopic myope, with an exophoria of 10° at reading distance. The fusion convergence required here for *each* eye is 5°. Physiological exophoria may be 2° for each eye, so that we have to deal with 3° of excess. Should we correct the whole of the excess with prisms? No, for we assume that since a myope employs greater convergence than accommodation, he has in consequence acquired greater power of fusion convergence than an emmetrope. Prisms

to relieve half the excess or even of 1° *d* each would probably meet the case. Exophoria occurs in other conditions of refraction than myopia, and in distant as well as in near vision.

When prisms are ordered to correct exophoria in near vision, care should be taken that they are not also used for distant vision, unless the exophoria for distance requires the same prisms. Esophoria may be present in distant vision, but, unlike exophoria, which is nearly always greater in near than in distant vision, tends to decrease in near vision, except in presbyopia, ciliary paresis, and hypermetropia. In the presence of esophoria, convergence needs to be *lessened* in order to see single. Now, therefore, the fusion convergence is negative instead of positive. In some of these cases convex lenses are indicated more than prisms. The latter when ordered should have their edges inwards.

Let us now consider the three grades of convergence.

Anatomical Position of the Eyes.—What is the *starting point* of convergence? We do not exactly know what position the eyes would take during life in the absence of any converging innervation, though the divergence which follows monocular amblyopia seems to show it would be one of considerable divergence. Were all the innervations to cease, the anatomical position of rest of the eyes would undoubtedly be one of

considerable divergence, as Hansen Grut ably maintained in his Bowman* lecture. Le Conte† showed that during sleep, and even, in his case, during drowsiness, the eyes diverge, as they also do in drunkenness, under chloroform, and at death. In ophthalmoplegia externa, divergence also occurs.

Tonic Convergence.—The ocular muscles no doubt possess a physiological tone, similar to that of the other skeletal muscles, but what influence it has on the position of the eyes it is difficult to decide. In addition to this common muscular tone there is a persistent activity of the converging innervation, which disappears when deeply under chloroform, but which in ordinary conditions prevents the eyes from assuming their position of anatomical divergence. George Berry views it as "the tendency to persistence of a constantly called for state of innervation," while Hansen Grut aptly compares it to the tonic element in the accommodation of a hypermetrope. By this tonic convergence, the visual axes are brought to practical parallelism, so that on viewing a distant object, and occluding one eye, it either remains undeviated, or only aberrates slightly. In *Fig.* 73 the strong lines (*R*, *R*) indicate the supposed position of the visual axes

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* *Trans. Ophth. Soc.*, Vol. x., p. 1.

† Le Conte on *Sight*, p. 254.

were all nervous impulse abolished. The lines *i*, *i* indicate the tonic convergence during waking hours, due partly to muscular tone, and partly to involuntary tonic action of the converging innervation. It is this position of the visual axes (*i i*), to which they are brought by tonic convergence, that is noticed when we use Von Graefe's "distant equilibrium test," or any of its successors, such as phorometers, the double prism or glass rods. Exophoria in distant vision indicates a deficiency, and esophoria an excess,

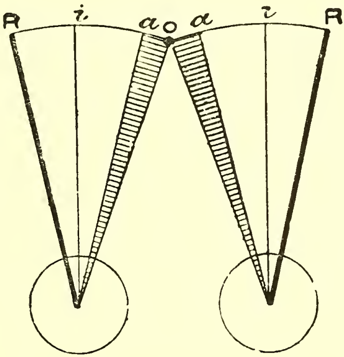

Fig. 73.—The three grades of convergence.

of *tonic convergence.* In either case the aberration is corrected, when both eyes are in use, by "reflex convergence," or "reflex divergence," as the case may be.

Excess of tonic convergence is sometimes apt to occur temporarily after prolonged use of the eyes in near work, also more permanently in a certain

proportion of those myopes, who hold their work near their eyes, and yet retain binocular fixation. It occurs, too, in most hypermetropes if uncorrected by lenses, and in a few even when corrected. If found to be persistent, and suspected to give rise to asthenopia, excess of tonic convergence may be reduced by exercise with the prism-verger, or failing that, be temporarily relieved by prisms with their edges in, of a strength that will only partially correct the anomaly.

Deficiency of tonic convergence, evidenced by exophoria in distant vision, is rather common, and in the author's experience has been sometimes due to reflexes through the fifth nerve, especially from carious teeth. Want of muscular tone in the system generally predisposes to it. It is, however, found besides in many who are free from either of these conditions, and often in myopes, especially those who read with one eye. It is to be regarded as merely a symptom, generally harmless. If associated with defective prism-convergence, as shown by the prism-verger, training of the prism-converging power is indicated as described in chapter XI. Failing this, it can be relieved by prisms with their edges out, to correct a small portion of the defect. It would be a useful nomenclature for prisms to designate those with edge in as *plus*, and those with edge out as *minus*, just as we speak of plus and minus

lenses. Thus $-3° d$ would mean a prism, deviating 3°, edge out, and $+5°$ P. would mean a prism with physical angle of 5°, edge in. It is easy to remember that plus conditions of convergence are relieved by plus prisms, and minus conditions of convergence by minus prisms. In normal eyes, plus prisms increase the convergence, and minus prisms lessen it; and convergence is a positive quantity.

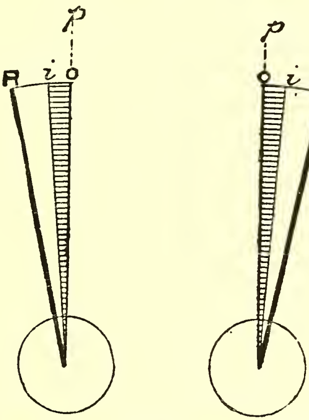

Fig. 74.—The two grades of convergence in distant vision. Tonic Convergence is *R i*, and Reflex Convergence is *i o*.

Accommodative Convergence. — In distant vision, as we have seen, there are only two grades of convergence, the tonic and the reflex, shown in *Fig.* 74. In near vision there is an intermediate grade—the accommodative. If one eye be occluded by the hand while vision is directed, first to a distant object, and then to a near

one, the occluded eye deviates inwards under the hand from an impulse to convergence, which is due chiefly to sympathy with accommodation, but also to the habit of converging when attention is directed to a near object (Grut's "Nahebewusstein"). The second grade of convergence is therefore added on to the first or tonic grade, and its amount depends, of course, on the amount of accommodation in exercise. As a rule, each dioptre of accommodation is accompanied by about three-quarters of a metre-angle of associated convergence, so that in a typical emmetrope, not presbyopic, the 4 D of accommodation in exercise for vision at a quarter of a metre, are accompanied by 3 m.a. of convergence, leaving a deficit of 1 m.a. to be made up reflexly, as shown in *Fig.* 72. To investigate the accommodative convergence at different distances, I have made scales, similar to that of *Fig.* 45, for use at 1m., ·5m., ·33m., and ·25m., but for practical purposes we need only use the last.

Now we may notice some of the conditions which *increase* the amount of accommodative convergence. Cycloplegia from any cause renders the ciliary muscle less responsive to its motor impulses, thereby necessitating increased impulse to accommodation, and with it an increase proportionately of the accommodative convergence.

If, even without such paralysis, the object

fixed be approached to the "punctum proximum" of accommodation, the accommodative effort becomes so much greater than the effect produced in the lens, that the associated convergence exceeds all proportion, and produces an esophoria. In hypermetropia, the accommodative convergence, without correction of refraction, is, of course, greater as a rule than in emmetropia.

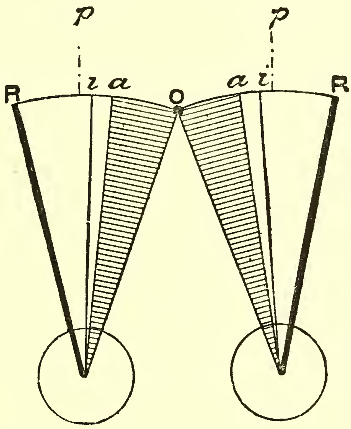

Fig. 75.—The conditions in a myope who has esophoria in distant with exophoria in near vision.

What conditions *lessen* the accommodative convergence? Any which make accommodation easier, so that work is done with less effort. Myopia, for instance, renders accommodation unnecessary in vision beyond the far point. *Figs.* 75 and 76 show how the diminution of accommodative convergence (*i a*) in myopia is

made up by greater reflex (and perhaps also voluntary) convergence (*a o*), the amount of which is influenced by the condition of the tonic convergence (*R i*), which is excessive in *Fig.* 75, and deficient in *Fig.* 76. To observe the effect of *eserine*, which, in weak solution, renders the ciliary muscle more excitable to stimulation without producing spasm or myopia, I placed

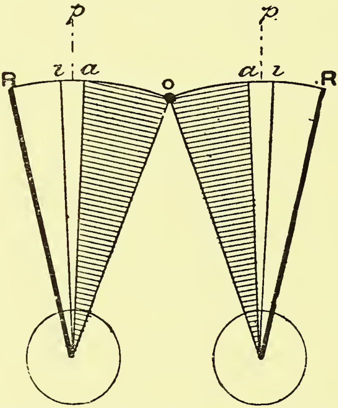

Fig. 76.—The conditions of a myope who has exophoria in distant and near vision.

gr. $\frac{1}{4000}$ within each lower lid. This allowed me to retain full distant vision, but caused exophoria at 10 inches, of from one to two metre angles. The effect on tonic convergence, as shown by the distant equilibrium, was practically *nil.* This experiment shows how truly convergence is affected, not by accommodation, but by accommodative impulse, for accommodation was the

same both with and without the eserine, but with it, the effort was less, and the impression on convergence reduced in consequence.

Fusion Convergence.—This, the third grade of convergence, has already been noticed in part. It is the element most affected by ocular fatigue, as is well seen in cases of "periodic squint," as they are called. On rising in the morning, there is generally no squint, but as the day wears on, and the eyes get tired, the squint appears, its manifestation being partly due to the diminishing vigour of the visual-reflex, the amplitude of which varies with the nervous vigour of the moment. As soon as the amplitude of the visual reflex becomes less than the squint, the latter is no longer overcome.*

Fig. 77 represents a case of true periodic squint in the early morning. The first two grades of convergence, each excessive if the patient be a hypermetrope, would bring the visual axes to *a a*, but they are brought back to *o* by negative reflex convergence (shaded area). As the day wears on the effort is abandoned, though in distant vision the squint may perhaps still be overcome. By correcting the hypermetropia we lessen the

* These cases must not be confused with other groups—such as "accommodative squints," in which the hypermetropia may be absolute, or accommodation is partially surrendered, except on paying close attention to an object, or cases in which the squint appears only in near vision, because accommodation there reaches the limits in which effort becomes disproportionate to work.

accommodative convergence, and the tonic convergence will thus gradually tend to get less, the excitability of the converging centre being

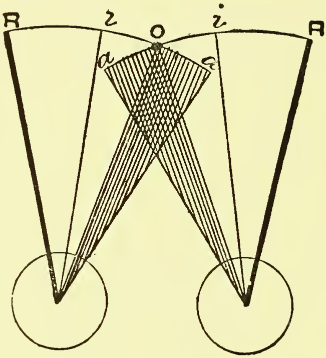

Fig. 77.—A periodic squint in its latent phase.

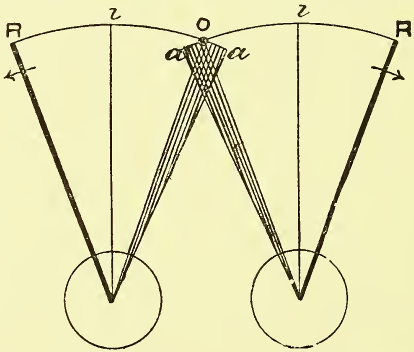

Fig. 78.—The same case as in *Fig.* 77, but relieved by tenotomy, which displaces the anatomical position outwards.

no longer kept up by stimulation from inordinate accommodation. It is thus that squints are so often cured by spectacles in the course of years.

The effect of tenotomy on the same case is shown in *Fig.* 78. By increasing the anatomical divergence it allows more room for the excessive tonic and accommodative convergences. The amplitude of the visual reflex can often be increased by nerve tonics, such as Fellowes' Syrup, rest, and change of air. Since the third grade of convergence is complementary to the

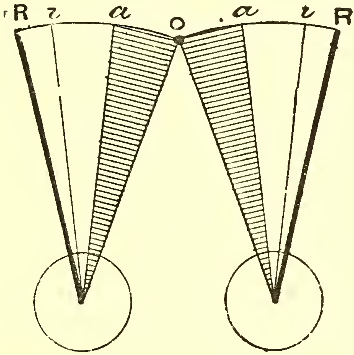

Fig. 79.—High fusion supplement (*a o*) from deficient tonic convergence (*R i*).

other two, its amount can be altered by altering them. In *Fig.* 79, for instance, the tonic convergence is deficient, the accommodative convergence normal, and the reflex convergence, therefore, excessive. The relief of this by prisms is shown in *Fig.* 80, and by tenotomy in *Fig.* 81. The prisms do not alter the anatomical position of the eyes (*R*, *R*), or alter the accommodative convergence, but they alter the position (*i*, *i*) which the eyes

assume in consequence of tonic convergence. We can alter

The *anatomical* position (R, R) by *tenotomy*,

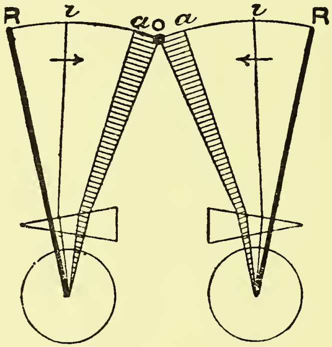

Fig. 80.—Same case as *Fig.* 79, relieved by prisms.

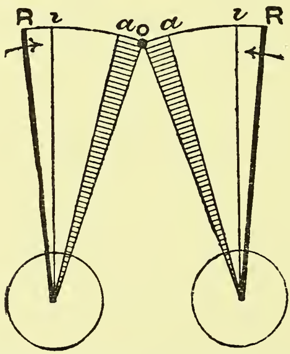

Fig 81.—Same case as *Fig.* 80, relieved by tenotomy.

The *tonic* position (i, i) by *prisms*,
The *accommodative* convergence by *lenses*,
The *fusion* convergence by any of the three.

Insufficient control of the converging centre is evidenced by strabismus.

Hypermetropia is the exciting cause of the ordinary converging strabismus. Its predisposing causes are various. In a small minority there is an undoubted defect in the cerebral fusion faculty. In nearly all the love of binocular vision is more or less enfeebled by inequality in the refraction, or in the visual acuteness of the two eyes, the squinting eye being that which is most hypermetropic or astigmatic.

When no such difference between the eyes exists, the squint is alternating, and the predisposing cause is either a defective fusion sense, or over-excitability of the converging centre, the insulation of which may, in such cases, be incomplete until later in life.

The theory that all squints are due to central fusion defect *only*, is, in the author's view, untenable. Were it true, all squints would date from birth, whereas it is a matter of common knowledge that a very large proportion date from the age of three years, after binocular vision has been enjoyed for some time. Moreover, they generally commence in near vision only, or when the attention is concentrated upon an object.

This proves that single vision is sufficiently appreciated to overcome gentle obstacles, but the faculty of fusion divergence is not sufficiently

developed to conquer the strong impulse to convergence which accompanies the excessive accommodative impulse in hypermetropia.

Mr. Worth has done excellent service in impressing the duty of recovering any commencing amblyopia as early in life as possible.

Divergent strabismus belongs in general to a later age, because its two chief causes are myopia, which does not commence until after early childhood, and increasing diminution in the excitability of the converging centre. It is on account of the latter phenomenon that an eye blinded in later life nearly always gradually deviates outwards. Anatomical peculiarities of the ocular muscles, such as imperfect development of the interni, account for a small proportion of cases of divergence.

Metre-Angles.—The proper unit of convergence is the metre-angle proposed by Nagel. It is the angle through which *each* visual line sweeps inwards from parallelism to a point one metre from the eyes and situated half-way between the visual lines when they were parallel.

This is the *first* metre-angle. The second is the angle swept through in glancing from the metre point to a half-metre point. The third from a half-metre point to a third-metre point. Successive metre-angles in this way are not of strictly equal magnitude, but it is usual to assume them equal to the first metre-angle for

convenience. The size of this, however, depends entirely on the distance between the centres of motion of the two eyes, which is called for convenience the interocular distance. In working with metre-angles the sine-centune affords the only absolutely exact unit.

The first metre-angle contains as many centunes as there are centimetres in half the inter-ocular distance. Thus with an interocular distance of $6\frac{1}{2}$ centimetres the first metre-angle will be $3\frac{1}{4}$ centunes. We have seen that a centune is about $\frac{4}{7}$ of a degree, so that conversion into degrees becomes very easy. For example, multiplying $3\frac{1}{4}$ by 4 and dividing by 7 gives us $1\frac{6}{7}°$. So simple is this process of conversion that an elaborate table of metre-angles is unnecessary.

In clinical work the metre-angle is rarely employed in this country, its interest being physiological.

The subject of *relative convergence* has already been discussed in an earlier chapter. It is the amount by which accommodation can be increased or relaxed during fixed accommodation for points at chosen distances, and is best measured by the prism-verger of *Fig.* 49, which for this purpose the author would prefer to have marked in centunes, so as to bear a simpler relation to metre-angles. There is no reason why it should not be graduated in both centunes and degrees.

CHAPTER XVIII

THE STRICT EFFECT OF PRISMS AND DECENTRED LENSES ON THE FIXATION LINES

It has hitherto been taken for granted that an eye before which a prism is placed suffers a deviation which is equal to the deviation of light by the prism. This, however, is not the case.

1. **As regards Prisms.**—A vertical prism, which fully corrects a hyperphoria in distant vision,

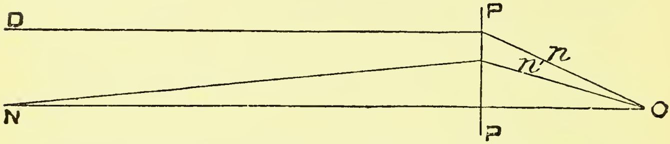

Fig. 82.—To show that the deflection of the eyeball by a vertical prism is less in near than in distant vision.

only partially corrects it in near vision, for its effect on the vertical separation of the visual axes is greater in distant than in near vision. This is shown in *Fig.* 82, where the line *O N* represents the visual axis of the naked eye, and the lines *n* and *n′* that of the successive positions of the other eye, on looking, through a prism, first at a distant and then at a near object.

For practical purposes, since the position of the

prism, and the direction of vision, are so inconstant, it will be better to make a simple approximate rule such as the following, rather than to give needlessly exact formulæ.

To find the amount by which the deflection of the eyeball is diminished in near vision, *divide the deviation of the prism by the distance in inches of the object from the eye.* Thus, with vision for five inches, the elevation of the visual axis by a prism is one-fifth less than in distant vision, so that a prism of 5° *d* would have an effect of 4° on the visual axis. With vision for ten inches, the elevation of the visual axis would be one-tenth less, and so on. It is well to be aware of this fact, because, if a patient fitted with vertical prisms says they suit him best in near vision, it may mean they are too strong, whereas, if they suit him as well in distant vision, they are probably not.

2. **As regards Decentred Lenses.**—In an essay,* written many years ago, it was shown by the author that the prismatic equivalent of decentring a lens, and the effect of that decentred lens on the visual line, are two distinct things. Suppose we decentre a lens, or, what is equivalent, suppose we place a normally centred lens precisely before one eye and associate a prism with it. It has hitherto been supposed that the angle of deviation of this prism would exactly express the effect on convergence. But it is not so.

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* *Syme Surgical Fellowship Essay,* 1884.

A pencil of light is deflected equally by + and − lenses of equal power, if they are equally displaced or decentred.

The visual line, however, is deflected more by the + lens and less by the − lens; so that, other things being equal—

Decentring a + lens has more effect than decentring a − lens; and

When prisms are combined with + lenses their effect is increased; when with − lenses their effect is lessened.

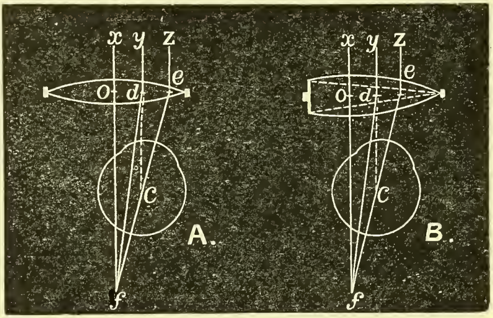

Fig. 83.—*A*. The right eye is looking at a distant object through a lens *displaced* inwards, *rim and all*, by a distance *o d*; *d c e* is the effect on convergence. *B* is the same eye looking through a lens of the same focal length, but *decentred in* its rim by the same distance *o d*. The dotted lines indicate the prism that would be equivalent to the decentring, and the effect on convergence is the same as with *A*.

Fig. 83 *B* represents a decentred lens, the action of which is precisely similar to a prism and lens, for, as said before, it is as if the lens were split in two, and the prism shown in dotted

outline were inserted between the two halves. Now it will be evident at a glance, that the angle *c d f* is the deviating angle of the virtual prism, while the greater angle, *d c e*, is that which represents the effect on convergence. It makes no difference to the effect on convergence whether a lens is decentred in its frame, as in *Fig. B*, or displaced, frame and all, to the same amount, as in *Fig. A*.

There is one exception to the discrepancy between the prismatic equivalent and the effect on convergence, and that is when the lenses are *convex* and the object of fixation is at their focal length, and immediately in front of the eye. Within that distance the effect on convergence is less, and beyond it more, than the prismatic equivalent. But with *concave* lenses it is always *less*. In the first edition I went no further than to point out these facts, though in a former paper* I had given formulæ for the effect of decentring on the visual axis in distant vision, and a series of simple rules for both near and distant vision, which may now be reproduced in a slightly altered form.

If a line (not represented in the figures) be dropped from *c* (the centre of rotation of the eye), perpendicularly upon the principal axis (*o f*) of the lens, the length of this perpendicular equals the decentration of the lens (*o d*). It also cuts off

* *Jour. Anat. and Phys.*, Vol. xxi., p. 36, 1884.

from the line $o\,f$, which measures the focal length of the lens, a portion equal to $d\,c$.

Therefore

$$\text{Tan. } o\,f\,e = \frac{o\,d}{o\,f - d\,c}.$$

But $o\,f\,e$ is equal to the angle $d\,c\,e$ which expresses the effect of the decentred lens on the visual axis, and which we may represent by x. Let d stand for the decentring ($o\,d$), and f for the focal length of the lens in mm. ($f\,o$), and let the distance of the lens from the centre of rotation of the eye be 25 mm. Then, with a convex lens

$$\text{Tan. } x = \frac{d}{f - 25}$$

and with a concave lens,

$$\text{Tan. } x = \frac{d}{f + 25}.$$

In these formulæ,* the distance of 25 mm. for the distance of the lens from the centre of the eye was chosen because, being about an inch, it allowed some simple rules to be made as follow. They are true to a practical approximation, neglecting spherical aberration, and the difference between tangents and arcs.

With a convex lens the increment is $\frac{1}{f-1}$; with a concave lens it is $-\frac{1}{f+1}$.

I.—*To find the effect on the visual axis of combining a lens and a prism, or of decentring a lens* (it being quite immaterial what prism-unit is

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* *Jour. Anat. and Phys.*, Vol. xxi., p. 35 (*Essay awarded Syme Fellowship*, 1884).

employed so long as the focal length of the lens is known in inches).

(*a*). *With a + lens. Rule: Divide the deviating angle of the prism* (in the case of decentring the *virtual* prism can be found from Table V.) *by the number of inches in the focal length of the lens* LESS ONE. This, *added* to the deviating angle of the prism, gives the effect on the axis of fixation. *Example:* Given a 3° *d* prism and a +10D lens, required the effect on the visual axis. A +10D lens has a focal length of four inches. This, *less one*, is three inches.

$$\frac{3^\circ}{3} = 1^\circ. \quad \text{Add this to } 3^\circ = 4^\circ.$$

The effect on the visual axis, therefore, is 4°.

(*b*). *With a − lens. Rule: Divide the deviating angle of the prism* (or of the virtual prism from Table V. in the case of decentring) *by the number of inches in the focal length of the lens* PLUS ONE. This, *deducted* from the deviating angle of the prism, leaves the effect on the axis of fixation. *Example:* Given a 3° *d* prism, and a − 10D lens, required the effect on the fixation line. The focal length of a − 10D lens is four inches, which *plus one* is five inches. The deviating angle must therefore be divided by 5, thus:

$$\frac{3^\circ}{5} = 36'; \ 3^\circ - 36' = 2^\circ\ 24'.$$

The effect on the visual axis is, therefore—2° 24′.

II.—*To find what prism we must combine with a lens, in order to produce any required effect on the axis of fixation.*

(*a*). *With a + lens. Rule : Divide the required effect on the fixation line by the number of inches in the focal length of the lens.* This, *deducted* from the effect required, gives the prism that will produce it. *Example :* Required with a + 10D lens to deflect a fixation line by 4°. What prism is needed ? The focal length of the lens is four inches, $4^\circ \div 4 = 1^\circ$. Deduct this from $4^\circ = 3^\circ$. The prism required is 3° *d*. Or the decentring required is (from Table V.) a trifle more than 5 mm., since a lens of 10D decentred to that amount has a prismatic equivalent of 3° *d*.

(*b*). *With a – lens. Rule : Divide, as before, the required deflection of the axis by the number of inches in the focal length of the lens.* This, *added* to the deflection required, gives the prism that will produce it. *Example* : Required with a – 10D lens to deflect a visual axis by 4°. What prism is needed ? The focal length is four inches. $4^\circ \div 4 = 1^\circ$. Add this to $4^\circ = 5^\circ$. The prism required is 5° *d*.

From Table V. it is seen that a lens of – 10D must be decentred nearly 9 mm. to have the effect of a prism of 5°.

These rules may prove of value in the correction of hyperphoria, where considerable exactness is required. The examples given illustrate well how

much less deflection is imparted to the line of fixation by decentring a concave lens than a convex one. Thus, we have seen that to produce a deflection of 4°, a convex 10D lens needs a decentring of 5 mm., and a corresponding concave lens 9 mm.

For near vision the rules require a little modification. Instead of reading the "focal length" of the lens we should read the "conjugate focus." They then apply in near vision with quite sufficient accuracy for practical purposes, for though it is true that they suppose the object of fixation to be in front of each eye, when it is really in the median plane, this only introduces a trifling error, and it must be remembered that no fixed rules or tables are quite accurate clinically, since the interocular distance, and the distance of the lenses from the eyes, vary so greatly in different patients.

To find the conjugate focus of a lens we *subtract* from the reciprocal of its focal length, the reciprocal of the distance of the object from the lens. Thus, with the object at 12 inches, a + 10D lens (whose focal length is therefore 4 inches) has a conjugate focus of ($\frac{1}{4} - \frac{1}{12} = \frac{1}{6}$) 6 inches.

Since a concave lens has a virtual focus, it is prefixed by the negative sign, and therefore we *add* the reciprocals; thus, with a − 10D lens under the same conditions as before, the conjugate focus is ($-\frac{1}{4} - \frac{1}{12} = -\frac{1}{3}$) 3 inches.

Some may perhaps find a difficulty in using these rules, and fortunately the subject has recently been followed up by Dr. Percival, who is an able mathematician, and has drawn up a series of tables from a formula which assumes reading distance to be $\frac{1}{3}$ metre, the distance between the ocular centres to be 64 mm., and the distance of the lenses from the centres of rotation of the eyes to be 27·4751 mm. This distance is chosen since it is that at which lenses should theoretically be placed in order that the size of images formed on the retina may be the same as in emmetropia : it is, in short, the distance of the anterior principal focal plane of the eye from the centre of rotation. They are made for distant vision from the formula, similar to my previous one ;

$$\text{Tan. } \chi = \frac{d}{f - k}$$

Where χ = deflection of visual axis.
f = focal length of lens
k = distance of lens from centre of rotation.

For near vision I had made no formula, but Dr. Percival has contributed one to supply this deficiency, namely—

$$\text{Tan. } \chi = \frac{d\,p - f\,m}{f\,p - k\,(p - f)}$$

This is for a convex lens ; for a concave one the signs must be reversed, being plus instead of minus, since a negative value must be given to f.

DR. PERCIVAL'S TABLE FOR DISTANT LENSES

TABLE VI

				1 D	2 D	3 D	4D	5 D	6 D	7 D	8 D	9 D	10 D	12 D	14 D	16 D	18 D	20 D
CONVEX.	Divergence decentration inwards. Convergence outwards.	4 ma.	7°17′	124·5 7°6′	60·5 6°54′	39·1 6°42′	28-5 6°30′	22·1 6°18′	17·8 6°6′	14·7 5°54′	12·5 5°41′	10·7 5°30′	9·3 5°18′	7·1 4°54′	5·6 4°30′	4·5 4°6′	3·6 3°42′	2·9 3°18′
	”	3 ma.	5°29′	93·3 5°20′	45·3 5°11′	29·3 5°2′	21·3 4°53′	16·5 4°44′	13·3 4°35′	11·1 4°26′	9·3 4°17′	8·0 4°7′	6·9 3°59′	5·3 3°41′	4·2 3°23′	3·3 3°5′	2·7 2°47′	2·1 2°29
		2 ma.	3°40′	62·2 3°33′	30·2 3°28′	19·6 3°21′	14·2 3°16′	11·0 3°9′	8·9 3°4′	7·4 2°57′	6·2 2°52′	5·3 3°45′	4·6 2°40′	3·6 2°27′	2·8 2°15′	2·2 2°3′	1·8 1°51′	1·4 1°39′
		1 ma.	1°50′	31·12 1°47′	15·121 1°44′	9·787 1°41′	7·121 1°38′	5·521 1′35′	4·454 1°32′	3·692 1°29′	3·121 1°26′	2·676 1°23′	2·321 1°20′	1·787 1°14′	1·406 1°8′	1·121 1°2′	·898 55′	·721 50′
			1° 0	16·975 58′	8·248 57′	5·339 55′	3·884 53′	3·011 52′	2·429 50′	2·014 48′	1·702 47′	1·460 45′	1·266 43′	·975 40′	·767 37′	·611 33′	·490 30′	·393 27′
CONCAVE.	Convergence decentration inwards. Divergence outwards.		1°	17·935 1°2′	9·207 1°3′	6·298 1°5′	4·843 1°6′	3·970 1°8′	3·389 1°10′	2·973 1°12′	2·661 1 °13′	2·419 1°15′	2·225 1°16′	1·934 1°20′	1·726 1°23′	1·570 1°26′	1·449 1°30′	1·352 1°33′
		1 ma.	1°50′	32·879 1°53′	16·879 1°56′	11·546 1°59′	8·879 2°2′	7·279 2°5′	6·212 2°8′	5·450 2°11′	4·879 2°14′	4·435 2°17′	4·079 2°20′	3·546 2°26′	3·165 2°32′	2·879 2°38′	2·657 2°44′	2·479 2°50′
	”	2 ma.	3°40′	65·75 3°45′	33·7 3°52′	23·1 3°57′	17·7 4°4′	14·5 4°10′	12·4 4°15′	10·9 4°22′	9·7 4°28′	8·8 4°34′	8·1 4°40′	7·1 4°52′	6·3 5°4′	5·7 5°16′	5·3 5°28′	4·9 5°36′
		3 ma.	5°29′	98·6 5°38′	50·6 5°47′	34·6 5°56′	26·6 6°5′	21·8 6°14′	18·6 6°23′	16·3 6°32′	14·6 6°41′	13·3 6°50′	12·2 6°59′	10·6 7°16′	9·5 7°24′	8·6 7°42′	7·9 8°10′	7·4 8°27′
		4 ma.	7°17′	131·5 7°29′	67·5 7°41′	46·2 7°53′	35·5 8°5′	29·1 8°17′	24·8 8°29′	21·8 8°41′	19·5 8°52′	17·7 9°4′	16·3 9°16′	14·2 9°39′	12·6 10°3′	11·5 10°26′	10·6 10°50′	9·9 11°12′

The object of observation is presumed to be at a distance of more than 6 metres from the patient.
The figures in larger type indicate the amount of decentration in millimetres.
The figures in smaller type represent the deviating power of the prisms whose action is equivalent to that of the decentration of the lenses.

DR. PERCIVAL'S TABLES FOR NEAR VISION

TABLE VII—(CONVEX)

			1 D	2 D	3 D	4 D	5 D	6 D	7 D	8 D	9 D	10 D	12 D	14 D	16 D	18 D	20 D
DIVERGING.	4 ma.	7°17′	240·6 13°32′	118·5 13°20′	77·8 13°8′	57·5 12°57′	45·3 12°46′	37·2 12°34′	31·3 12°22′	27·0 12°11′	23·6 11°59′	20·9 11°48′	16·8 11°26′	13·9 11°2′	11·7 10°47′	10·0 10°21′	8·7 9°51′
	3 ma.	5°29′	206·6 11°40′	101·9 11°31′	67·1 11°23′	49·7 11°14′	39·2 11°0′	32·2 10°57′	27·2 10°48′	23·5 10°39′	20·6 10°31′	18·3 10°22′	14·8 10°4′	12·3 9°47′	10·4 9°29′	9·0 9°11′	7·8 8°54′
	2 ma.	3°40′	172·6 9°47′	85·4 9°42′	56·3 9°36′	41·8 9°30′	33·1 9°24′	27·3 9°18′	23·1 9°12′	20·0 9°7′	17·6 9°1′	15·7 88°55′	12·8 8°43′	10·7 8°31′	9·1 8°17′	7·9 8°7′	6·9 7°56′
	1 ma.	1°50′	138·6 7°53′	68·8 7°50′	45·6 7°48′	34·0 7°45′	27·0 7°42′	22·3 7°39′	19·0 7°36′	16·5 7°33′	14·6 7°30′	13·1 7°27′	10·7 7°21′	9·1 7°15′	7·8 7°9′	6·9 7°3′	6·1 6°57′
		0	104·6236	52·3118	34·8745	26·1559	20·9247	17·4372	14·9462	13·0779	11·6248	10·4623	8·7186	7·4731	6·5389	5·8124	5·2312
CONVERGING.			5°58′	5°58′	5°58′	5°58′	5°58′	5°58′	5°58′	5°58′	5°58′	5°58′	5°58′	5°58′	5°58′	5°58′	5°58′
	1 ma.	1°50′	70·6 4°2′	35·7 4°5′	24·1 4°8′	18·3 4°11′	14·8 4°14′	12·5 4°18′	10·8 4°21′	9·6 4°23′	8·6 4°26′	7·8 4°29′	6·7 4°36′	5·8 4°41′	5·2 4°47′	4·7 4°53′	4·3 4°59′
	2 ma.	3°40′	36·6 2°6′	19·2 2°12′	13·4 2°18′	10·5 2°24′	8·7 2°30′	7·6 2°36′	6·7 2°42′	6·1 2°48′	5·6 2°54′	5·2 3°	4·6 3°12′	4·2 3°24′	3·9 3°36′	3·7 3°48′	3·5 4°
	3 ma.	5°29′	2·6 9′	2·6 18′	2·6 27′	2·6 36′	2·6 45′	2·6 54′	2·6 1°3′	2·6 1°13′	2·6 1°22′	2·6 1°31′	2·6 1°49′	2·6 2°7′	2·6 2°25′	2·6 2°43′	2·6 3°1′
	4 ma.	7°17′	−31·3 (1°48′)	−13·9 (1°36′)	−8·1 (1°24′)	−5·2 (1°12′)	−3·4 (59′)	−2·3 (47′)	−1·4 (35′)	−·84 (23′)	−·36 (11′)	·03 1′	·61 25′	1·02 49′	1·3 1°13′	1·58 1°37′	1·77 2°2′
Difference for 1 ma			33·995 1°57′	16·558 1°54′	10·743 1°51′	7·839 1°48′	6·0957 1°45′	4·933 1°42′	4·1029 1°39′	3·4801 1°36′	2·9957 1°33′	2·6082 1°30′	2·0270 1°24′	1·6119 1°18′	1·3004 1°12′	1·0582 1°6′	·8645 59′
Difference for 1°			18·54 1°3′	9·032 1°2′	5·861 1°	4·276 59′	3·325 57′	2·691 55′	2·2380 53′	1·8983 52′	1·6341 50′	1·4227 49′	1·1057 46′	·8792 42′	·7093 39′	·5772 36′	·471 32′

The object of observation is presumed to be $\frac{1}{3}$ metre from the centre of rotation of the globe.
The figures in larger type give the amount of decentration in millimetres.
$-^{ve}$ sign indicates decentration outwards; $+^{ve}$ sign decentration inwards.
The figures in smaller type represent the deviating power of the prisms whose action is equivalent to that of the decentration of the lenses. When enclosed in brackets the prisms are adducting in function, and should be placed edges inwards.

DR. PERCIVAL'S TABLES FOR NEAR VISION

TABLE VIII—(Concave)

			−1 D	−2 D	−3 D	−4 D	−5 D	−6 D	−7 D	−8 D	−9 D	10 D	−12 D	−14 D	−16 D	−18 D	−20 D
Diverging.	4 ma.	7°17′	−247·6 13°55′	−125·6 14°6′	−84·9 14°17′	−64·5 14°28′	−52·3 14°40′	−44·2 14°51′	−38·4 15°2′	−34·0 15°14′	−30·6 15°25′	−27·9 15°36′	−23·8 15°59′	−20·9 16°21′	−18·8 16°43′	−17·1 17·5′	−15·7 17°27′
	3 ma.	5°29′	−211·9 11°58′	−107·2 12°7′	−72·4 12°15′	−54·9 12°24′	−44·5 12°32′	−37·5 12°41′	−32·5 12°49′	−28·8 12°58′	−25·9 13°7′	−23·5 13°16′	−20·1 13°33′	−17·6 13°50′	−15·7 14°7′	−14·3 14°24′	−13·1 14°41′
	2 ma.	3°40′	−176·1 9°59′	−88·9 10°5′	−59·9 10°11′	−45·3 10°17′	−36·6 10°23′	−30·8 10°29′	−26·7 10°35′	−23·5 10°40′	−21·1 10°46′	−19·2 10°52′	−16·3 11·4′	−14·2 11°15′	−12·6 11°27′	−11·4 11°39′	−10·5 11°50′
	1 ma.	1°50′	−140·4 7°59′	−70·6 8°2′	−47·4 8°5′	−35·7 8°8′	−28·8 8°11′	−24·1 8°14′	−20·8 8°17′	−18·3 8°20′	−16·4 8°23′	−14·8 8°26′	−12·5 8°32′	−10·8 8°38′	−9·6 8°44′	−8·6 8°50′	−7·8 8°56′
		0	−104·6236 5°58′	−52·3118 5°58′	−34·8745 5°58′	−26·1559 5°58′	−20·9247 5°58′	−17·4372 5°58′	−14·9462 5°58′	−13·0779 5°58′	−11·6248 5°58′	−10·4623 5°58′	−8·7186 5°58′	−7·4731 5°58′	−6·539 5°58′	−5·8124 5°58′	−5·2312 5°58′
Converging.	1 ma.	1°50′	−68·9 3°56′	−34·0 3°53′	−22·4 3°50′	−16·5 3°47′	−13·1 3°44′	−10·7 3°41′	−9·1 3°38′	−7·8 3°35′	−6·9 3°32′	−6·1 3°29′	−4·9 3°23′	−4·1 3°17′	−3·5 3°11′	−3·0 3°5′	−2·6 2°59′
	2 ma.	3°40′	−33·1 1°54′	−15·7 1°48′	−9·8 1°42′	−6·9 1°36′	−5·2 1°30′	−4·0 1°24′	−3·2 1°18′	−2·6 1°12′	−2·1 1°5′	−1·7 59′	−1·1 47′	−·73 35′	−·42 23′	−·18 11′	−·01 (1′)
	3 ma.	5°29′	2·6 (9′)	2·6 (18′)	2·6 (27′)	+2·6 (36′)	2·6 (45′)	2·6 (54′)	2·6 (1°3′)	2·6 (1°12′)	2·6 (1°21′)	2·6 (1°31′)	2·6 (1°49′)	2·6 (2°7′)	2·6 (2°25′)	2·6 (2°43′)	2·6 (3°1′)
	4 ma.	7°17′	38·4 (2°12′)	20·9 (2°24′)	15·1 (2°36′)	12·2 (2°48′)	10·5 (3°)	9·3 (3°12′)	8·5 (3°24′)	7·9 (3°36′)	7·4 (3°48′)	7·0 (4°)	6·4 (4°24′)	6·0 (4°48′)	5·7 (5°12′)	5·4 (5°36′)	5·2 (6°)
Difference for 1 ma.			35·754 2°3′	18·316 2°6′	12·504 2°9′	9·598 2°12′	7·854 2°15′	6·6916 2°18′	5·8613 2°21′	5·2385 2°24′	4·7541 2°27′	4·3667 2°30′	3·7854 2°36′	3·3702 2°42′	3·0588 2°48′	2·8167 2°54′	2·6229 3°
Difference for 1°			19·502 1°7′	10·088 1°9′	6·820 1°10′	5·235 1°12′	4·284 1°14′	3·650 1°15′	3·1971 1°17′	2·8574 1°18′	2·5932 1°20′	2·3819 1°22′	2·0648 1°25′	1·8383 1°28′	1·6685 1°32′	1·5364 1°35′	1·4307 1°38′

The object of observation is presumed to be $\frac{1}{3}$ metre from the centre of rotation of the globe.
The figures in larger type give the amount of decentration in millimetres.
−ve sign indicates decentration outwards; +ve sign decentration inwards.
The figures in smaller type represent the deviating power of the prisms whose action is equivalent to that of the decentration of the lenses. When enclosed in brackets the prisms are abducting in function, and should be placed edges inwards.

It will be noticed that in the near vision table the prismatic equivalents differ from those of the distant vision table by a constant quantity, viz., 5′ for each degree, and 10′ for each metre-angle. I give Dr. Percival's own examples to illustrate their use.*

EXAMPLES ILLUSTRATING THE USE OF THE TABLES.

TABLE VI.—A hypermetrope of +8D and esophoria 1 ma. at 6 metres will have this convergence defect corrected by decentring the 8D lens 3·1 – mm. outwards, or, what comes to the same thing, by associating it with a prism of 1° 26′ *d* (not 1° 50′).

TABLE VII.—A patient requiring +12D glasses for reading, who can only maintain convergence for a distance of $\frac{1}{2}$ metre (2 ma.), must have his glasses decentred 4·6 mm. inwards.

TABLE VIII.—A myope requiring –6D for reading, who can only maintain 2 ma. of convergence, must have his glasses decentred 4 mm. outwards, or combined with a prism of 1° 24′ *d*, which is practically the same thing. Table VII.

*It has occurred to the author during revision of this last edition, that the omission to take account of spherical aberration unfortunately renders these tables only approximately accurate. They depend on the assumption that a beam of light filling the whole of a large lens is gathered to a single point, which, of course, is far from the case. In a strong lens we should only neglect spherical aberration near the principal axis.

and Table VIII. are useful also in estimating the relative range of convergence for reading distance. A myope using -5D for reading can obtain binocular vision with a $3°$ prism held edge inwards before each eye, as well as with a $12° \, 32'$ prism held edge outwards. His relative range is not, however, represented by $15° \, 32'$, but by $12° \, 46'$, or 7 ma.

APPENDIX

Sines, Arcs, and Tangents.—Should any reader be unacquainted with even the simplest elements of trigonometry, he will find no difficulty in understanding what follows.

Given any angle, select a point on one limb. On which limb, and at what distance from the apex, the point is chosen, is immaterial. Drop a perpendicular from this point on to the other

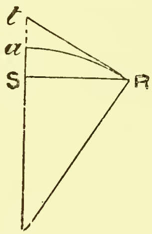

Fig. 84.—To illustrate the sine, arc, and tangent of an angle.

limb. Then, this perpendicular is to the longer limb as the sine of the angle; and is to the shorter limb as the tangent of the angle; while the shorter limb is to the longer limb as the cosine of the angle. *Fig.* 84 shows the relation which the arc bears to the sine and tangent. Let any angle be included between the two radii of a circle, then the portion of the circumference

included between the radii, as *a R* in the figure, is the *arc* of the angle, and, if the radius be taken as unity, a perpendicular (*RS*) dropped from the extremity of one radius perpendicularly upon the other radius is the *sine*, and a line drawn perpendicularly from the extremity of one radius to meet the other prolonged, as *R t* in the figure, is the *tangent* of the angle. It will be seen at once that the *sine* is always less, and the *tangent* always greater, than the *arc*, and the greater the angle,

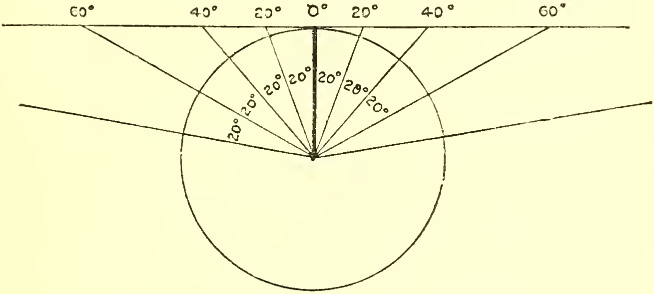

Fig. 85.—To illustrate a tangent scale.

the greater is the disproportion between the three quantities. *Fig.* 85 shows how, when angles are projected on to a straight line, as in the tangent scales represented in *Figs.* 44, 45, and 47, the graduations increase in magnitude with each departure from zero. The accompanying table (Table IX) of natural sines and tangents is inserted only for convenience, to save reference to a book of mathematical tables which may not always be at hand. The column of sines

TABLE IX (SINES AND TANGENTS)

Degrees.	Sines.	Tangents.	Degrees.	Sines.	Tangents.
1°	·01745	·01745	23°	·3907	·4245
2°	·0349	·03492	24°	·4067	·4452
3°	·05233	·0524	25°	·4226	·4663
4°	·06975	·0699	26°	·4384	·4877
5°	·08715	·0874	27°	·4540	·5095
6°	·10452	·1051	28°	·4695	·5317
7°	·12187	·1228	29°	·4848	·5543
8°	·13917	·1405	30°	·5	·5773
9°	·15643	·1584	31°	·5150	·6009
10°	·17364	·1763	32°	·5299	·6249
11°	·19081	·1944	33°	·5446	·6494
12°	·2079	·2125	34°	·5592	·6745
13°	·2249	·2309	35°	·5736	·7002
14°	·2419	·2493	36°	·5878	·7265
15°	·2588	·2679	37°	·6018	·7535
16°	·2756	·2867	38°	·6157	·7813
17°	·2924	·3057	39°	·6293	·8098
18°	·3090	·3249	40°	·6428	.8390
19°	·3256	·3443	41°	·6560	·8693
20°	·3420	·3640	42°	·6691	·9004
21°	·3584	·3839	43°	·6820	·9325
22°	·3746	·4040	44°	·6946	·9657

may be used for resultant prisms, and the column of tangents for making a tangent scale. For accurate working out of formulæ a fuller table must of course be used.

Formulæ for Refraction through Prisms.— In *Fig.* 86, a ray of light ($e\ f$) is seen traversing a

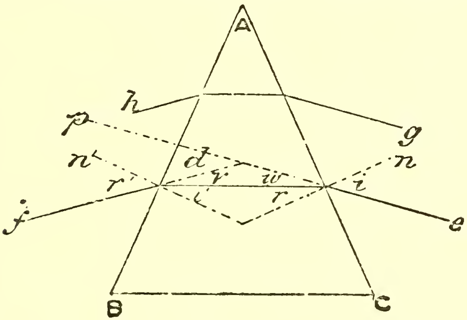

Fig. 86.

prism in the plane of a principal section. Then, whatever be the angle of incidence (i), the total deviation (d) is the sum of the deviations at the two surfaces, which are w and v respectively, so that

$$(1)\quad d = w + v.$$

Moreover, since n and n' are normals to the refracting surfaces,

$$(2)\quad A = r + i'.$$

But $\qquad w = i - r$

and $\qquad v = r' - i'$

so that by substitution,

$$(3)\quad d = i + r' - A.$$

From this we learn that however obliquely light

may traverse a prism, its total deviation is equal to the combined angles of its incidence and emergence less the edge angle of the prism.

To come to a special case. If the ray pass in the direction of *minimum deviation*, so that $i = r'$, then from (3)

$$(4) \quad d = 2i - A$$

and

$$(5) \quad i = \frac{d + A}{2}.$$

This last formula tells us that if we wish to pass a ray through a prism in the direction of minimum deviation, we must allow an angle of incidence equal to half the sum of the deviating and edge-angles.

Moreover, under the same conditions,

$$r = \frac{A}{2}$$

and since, by the law of sines, if we let μ stand* for the index of refraction,

$$\sin. i = \mu \sin. r.$$

$$(6) \quad i = \sin.^{-1}\left(\mu \sin. \frac{A}{2}\right)$$

Substituting this value for i in formula 4 we get

$$(7) \quad \sin d = 2\left(\mu \sin. \frac{A}{2}\right) - A.$$

This formula enables us to find the deviating

---

*The proper English symbol for the refractive index is μ. The introduction of the continental n is to be deprecated, being already appropriated for other purposes in mathematics.

angle of any prism, given its refracting angle (A) and its index of refraction (μ).

If we treat angles and their sines as equivalents, which we may do without serious error in the case of very weak prisms, then (7) becomes

$$(8) \quad d = A\,(\mu - 1).$$

This useful formula has been utilized on p. 22.

Lastly, if we wish to find the index of refraction of any transparent material, we can do so as follows.

By the law of sines,

$$(9) \quad \mu = \frac{\sin.\, i}{\sin.\, r}.$$ And since

$$(\text{by } 5) \quad i = \frac{d + A}{2}, \text{ and } r = \frac{A}{2},$$

$$(10) \quad \mu = \frac{\sin.\, \frac{A + d}{2}}{\sin.\, \frac{A}{2}}$$

From this formula, after finding the physical angle (A) of any prismatic piece of glass by Wollaston's reflecting goniometer or one of its modifications, and the deviating angle (d) by the methods described on p. 32, we can readily discover the index of refraction. Images by double internal reflection could, the author suggests, be utilized with advantage for this purpose through a formula modified from that given on p. 50.

Spirit-level Prismeter.—The idea, which occurred to the author, of measuring prisms by a spirit-level so as to render their faces horizontal in succession, lends itself more easily to "home-made" methods than the ordinary machine-made goniometer, since it rectifies more of its own errors.

A rod, say one metre long (or half if preferred), is hinged at one end to a piece of wood clamped to the end of the bench, while the other end rests on a wedge. A small block of wood, about two inches high and with a flat top one inch or more square, is fixed to the bar either near the hinge, or, should the bar be very long, about its middle. The free end of the rod is furnished with a metal tongue at its lower corner turning downward, while a metal strip projects beyond its upper corner. An ordinary centimetre scale is now erected at exactly one metre (or half a metre as the case may be) from the hinge, so that the metal strip can work against its face.

To adjust the scale, lay a spirit-level on the little platform, and when it reads horizontal set the zero of the scale against the metal strip of the bar once for all.

Any prism thereafter laid on the platform and covered by the same spirit-level would be measured by pressing in the wedge till the level again reads horizontal. Each centimetre on the vertical scale then corresponds to one prism-dioptre

(= % tan), or to two P.D. if the bar be half a metre long.

For finer work, as with a view to estimate the refractive index, the measurement is made rather differently. The wedge should be cut so as to rise one inch for every ten inches along the bench. For example, a wedge 20 inches long would stand 2 inches higher at its thick end than at the other, and it is well to affix a plane strip of metal to its upper face to act as a "road." A metal rule with its smooth side uppermost answers quite well.

A millimetre scale is affixed to the side of the wedge along its lower border with its zero near the apex. After again placing the level on the platform, adjust the wedge until the level reads horizontal. Now draw a cross line on the bench, or, if preferred, on a sheet of paper fixed to the bench by drawing pins, and lying under the wedge, exactly opposite the zero of the millimetre scale.

Place the prism to be measured on the platform, taking care that its base-apex line lies truly, and upon it the spirit-level, taking care that the same part of the level is used, and push in the wedge until the level again reads horizontal. Each *centimetre* of the scale that travels over the transverse line on the bench raises the end of the bar one millimetre and answers to what we might call a sine-millune, that is to a departure of 1 in 1000,

measured not along the tangent line, but along the sine line, see *Fig.* 87.

The angle of the prism therefore can be found from a table of sines, after setting down the number of millimetres travelled by the wedge and advancing the decimal point by 4 places. Each millimetre thus represents a sine value of ·0001.

To give an example, a prism which requires the wedge to be moved 174·5 mm. would possess an edge angle whose sine is ·01745, and it is therefore

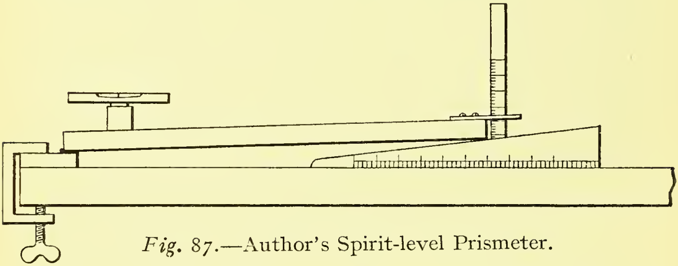

Fig. 87.—Author's Spirit-level Prismeter.

an angle of 1°. For strong prisms the wedge must be raised on blocks of wood of measured thickness, each centimetre in the thickness answering to ten centimetres of the scale on the wedge.

Instead of a plane top to the little platform it is sometimes better to furnish it with two parallel ridges of metal with straight edges, or one ridge and a metal point. These enable ringed prisms to be measured without taking off their rings. Moreover a little guard on the side of the platform towards the hinge is useful to keep the prism from slipping off.

Workman's Page.

A.—When simple Prisms are ordered.

(*a*) Treat centrads (▽), prisms-dioptres (△), and percentage units or centunes (%) all the same, each being $\frac{4}{7}$ of a degree, and each deflecting light by one centimetre at the distance of a metre. (Centune is a name in common for these units.) Confirm the prism by a chart like that on page 32, but with uniform intervals between the figures of as many centimetres as there are metres in the distance of the scale from the observer's most convenient standpoint; or use Prentice's "prismometric scale."

If only "degree prisms" are stocked, varying in accuracy, reckon a centune equal to a large degree of edge-angle, and to a half a large degree of deviation.

(*b*) When degrees are ordered note whether they refer to the edge-angle (Nos. 1, 2, etc.), or to the deviation (1° *d*, 2° *d*, etc.). The deviation may be taken as half the edge-angle. Confirm it by a scale of degrees as on page 32.

B.—When Prisms are ordered combined with Lenses (Sphero-Prisms).

After adding spheres to the prisms, make the fine adjustment by Prentice's plan on page 120, *or* Geneva prism measure, page 124, *or* by decentring method of page 122, after finding decentration that answers to prism from table on page 123.

C.—When Decentring is Ordered.

If slight, decentre the lens by the plan on page 122 (pure decentration). If more than size of lens allows, find the strength of prism that corresponds to decentring from table V, which gives the deviating angle. This, of course, needs doubling for the edge-angle. After grinding spheres on the prism selected, make fine adjustment as under *B*.

INDEX.

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